Methods in the Stability and Control of Periodically-Forced Aerospace Vehicles

Pennsylvania State University

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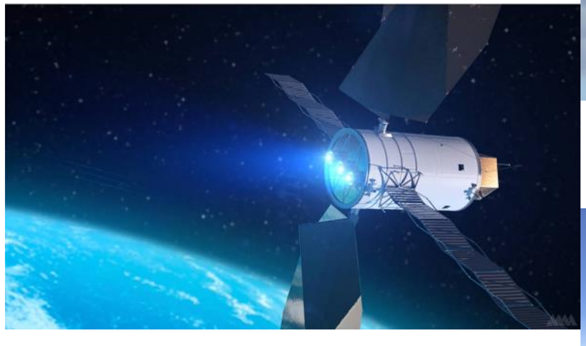
- **Introduction**
  - ❑Background
  - ❑Motivation
  - ❑Objectives
- **Methodology**
  - ❑Problem Definition
  - ❑Numerical Methods
- **Results**
  - ❑Flapping-Wing MAV
  - ❑Helicopter
  - ❑Flapping-Tail Airplane
- **Conclusions**
  - ❑Broader Implications
  - ❑Future Directions

Background

- **Periodically-forced systems** exist across multiple engineering disciplines
  - ❑Biological flyers (insects, birds, fish)
  - ❑Rotorcraft
  - ❑Spacecraft
  - ❑Wind turbines
- Dynamics represented by **Non-Linear Time-Periodic Systems (NLTP)**
- Stability analysis is challenging
  - ❑ Equilibrum represented by periodic orbit
- Two main approaches to stability analysis
  1. **Averaging methods**
  2. **Floquet theory**

Biological Flyer

Rotorcraft



Spacecraft

Wind Turbine

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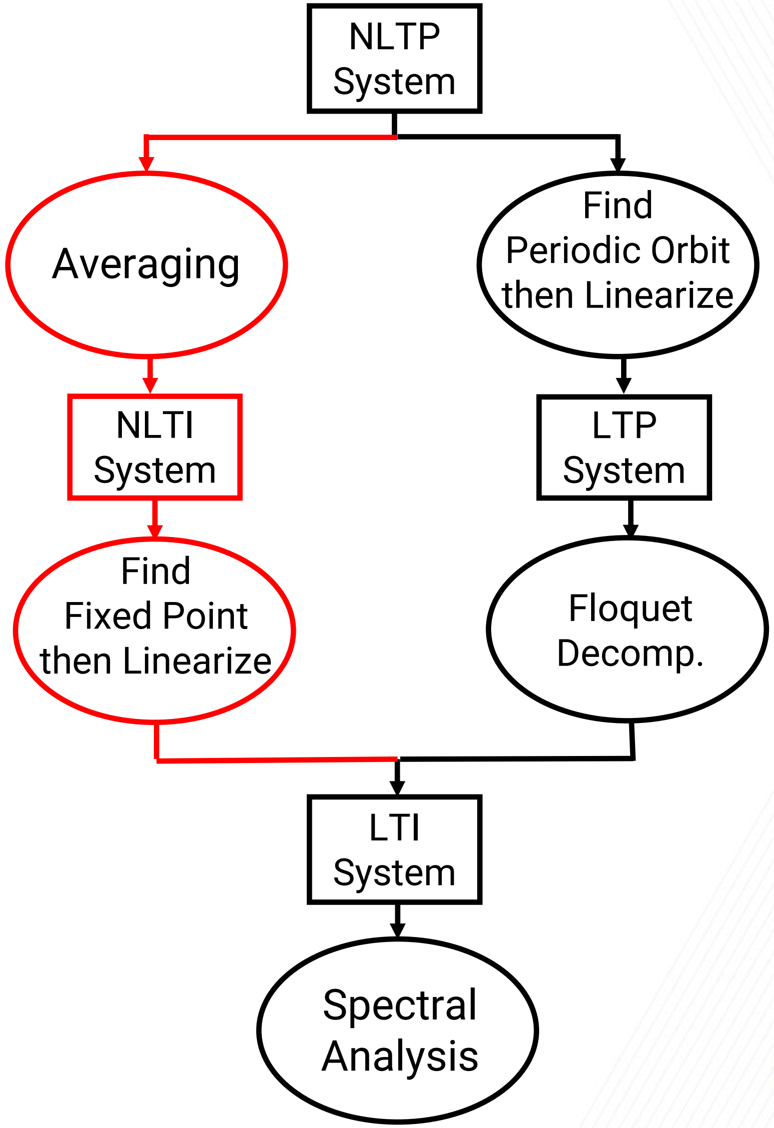
Background

Averaging methods

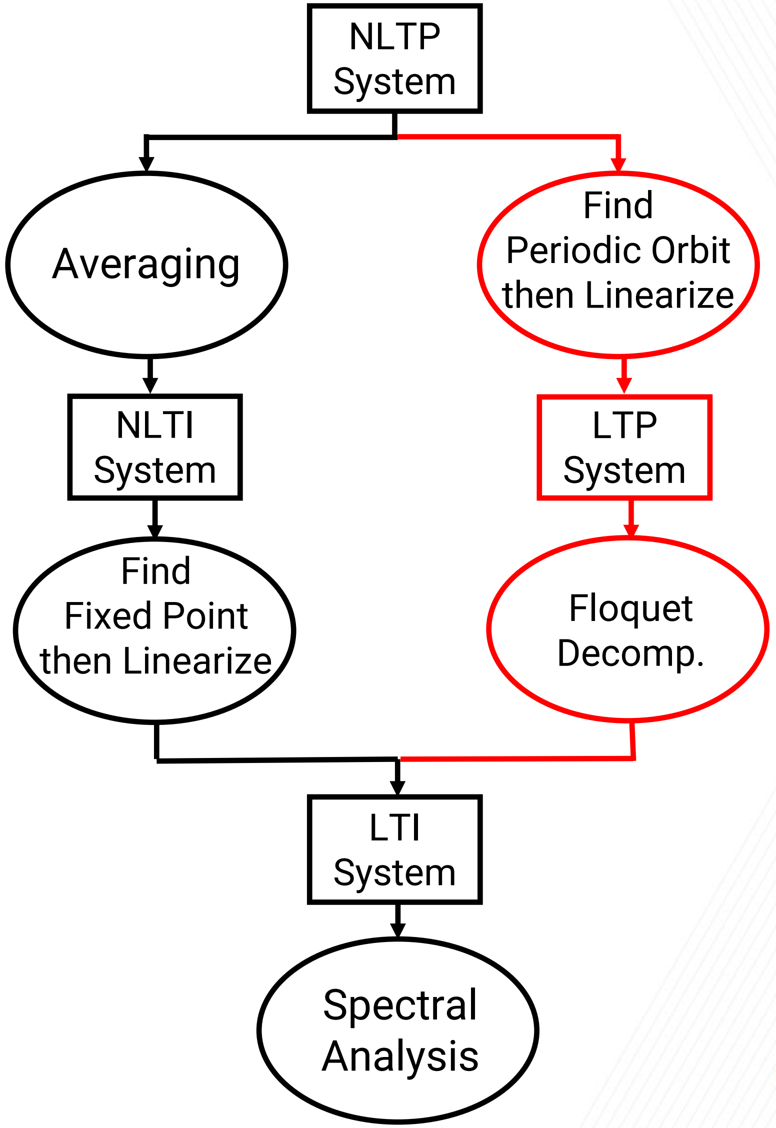
- Stability analysis
    1. Transform NLTP system into a **Non-Linear Time-Invariant (NLTI)** system
    2. Linearize about equilibrium point to yield **Linear Time-Invariant (LTI)** system
    3. Assess stability with spectral analysis
- Avoid direct calculation of periodic orbit



Background

Floquet theory

1. Need to solve for periodic orbit (i.e. **trimming**)
   - ❑Time marching (stable systems)
   - ❑Autopilot trim
   - ❑Periodic shooting
   - ❑Harmonic balance
2. Linearize about periodic orbit to yield **Linear Time-Periodic (LTP)** system
3. Transform system into equivalent LTI system (Floquet Decomposition)
4. Assess stability with spectral analysis



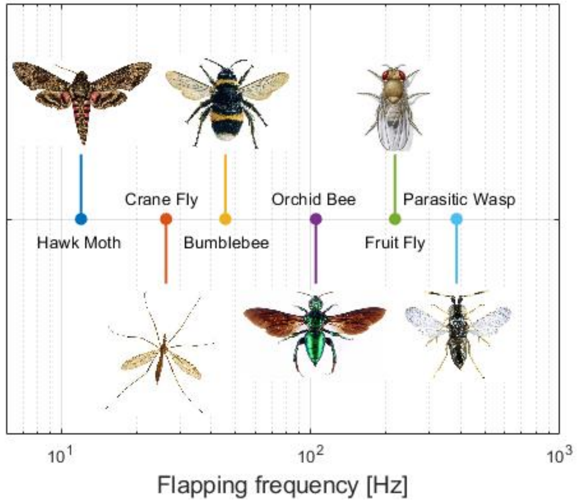
Motivation

Averaging methods

- Need time scale separation between forcing frequency and vehicle dynamics
- Not suitable for all systems
  - ❑ Birds, large-scale insects
  - ❑ Helicopters
- Cannot deal with non-smooth dynamics

Floquet theory

- Need for state transition matrices
  - ❑ Computationally intensive
  - ❑ Numerically sensitive
- Trimming methods
  - ❑ Time marching → stable systems
  - ❑ Autopilot trim → prior knowledge of dynamics
  - ❑ Periodic shooting → cannot solve for HHC input
  - ❑ Harmonic balance → state transition matrices



Flapping Frequency for Several Biological Flyers

UH-60 Black Hawk

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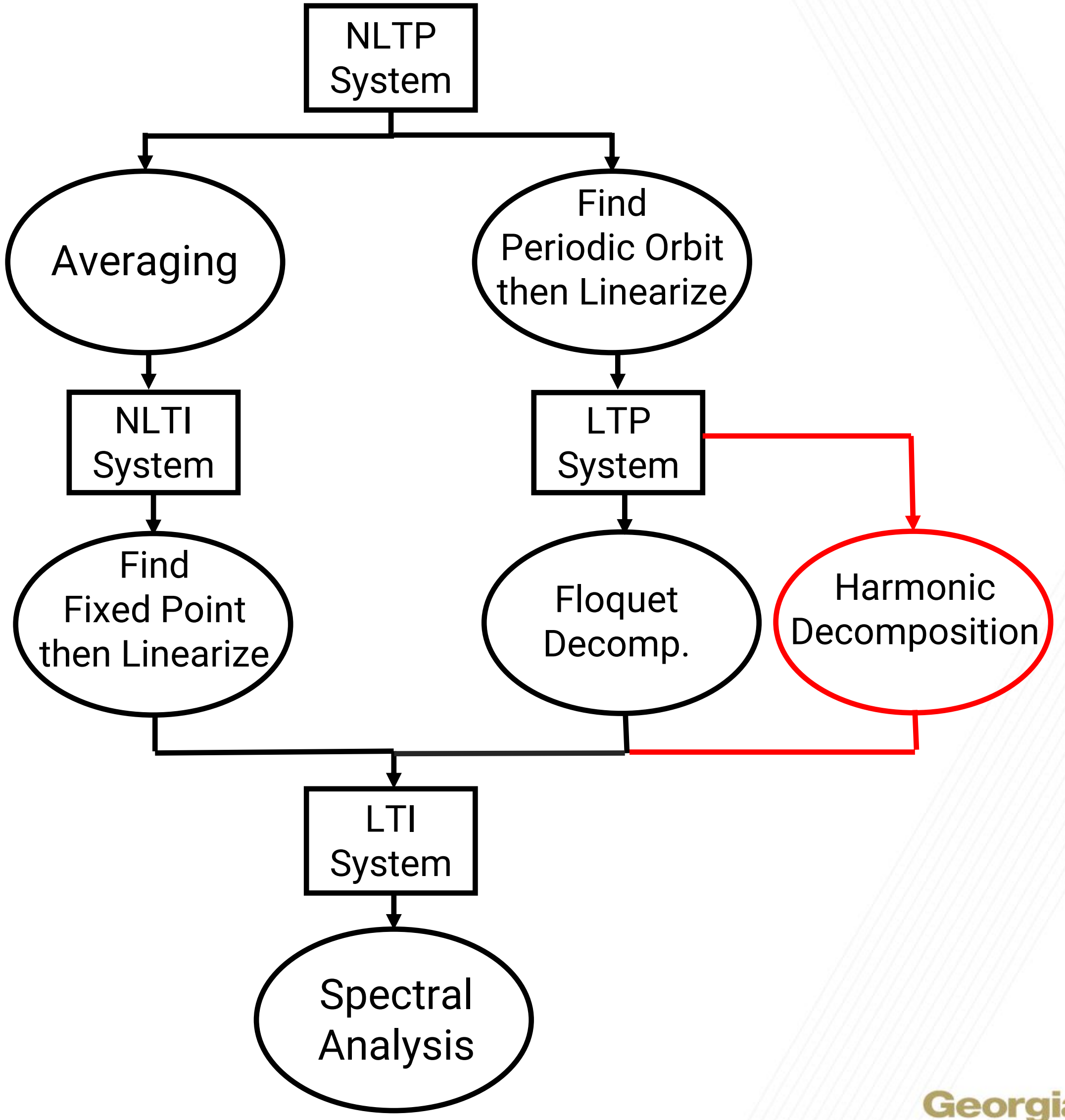
Motivation

Harmonic Decomposition

- First proposed for rotorcraft applications [Prasad et al. 2009]
- Used for approximating LTP systems with higher-order LTI models
- Does not rely on state transition matrices
- Numerically robust
- Could be used as alternative to Floquet Decomposition



Objectives

1. Develop an alternative approach to **stability analysis, and control design** of periodically-forced aerospace vehicles:
    - ❑ Does not rely on state transition matrices
    - ❑ Numerically robust
    - ❑ Based on harmonic decomposition
    - ❑ Can be used to compute the harmonic control inputs that attenuate arbitrary state harmonics
2. Demonstrate the approach on several periodically-forced aerospace vehicles
    - ❑ Flapping-wing Micro Aerial Vehicle (MAV)
    - ❑ Helicopter
    - ❑ Flapping-tail concept airplane

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Problem Definition

- Consider NLTP system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t)$$

  where
  - ❑ $\boldsymbol{x}(t) \in \mathbb{R}^n$ state vector
  - ❑ $\boldsymbol{u}(t) \in \mathbb{R}^m$ control vector
- Nonlinear dynamics $\boldsymbol{f}$ is T-periodic in t such that

$$\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t + T)$$

- Let $\boldsymbol{x}^*(t)$ and $\boldsymbol{u}^*(t)$ represent a periodic solution of the system such that

$$\boldsymbol{x}^*(t) = \boldsymbol{x}^*(t + T)$$
$$\boldsymbol{u}^*(t) = \boldsymbol{u}^*(t + T)$$

- Then, balance problem is stated as follows: determine $\boldsymbol{x}^*(t)$ and $\boldsymbol{u}^*(t)$ such that

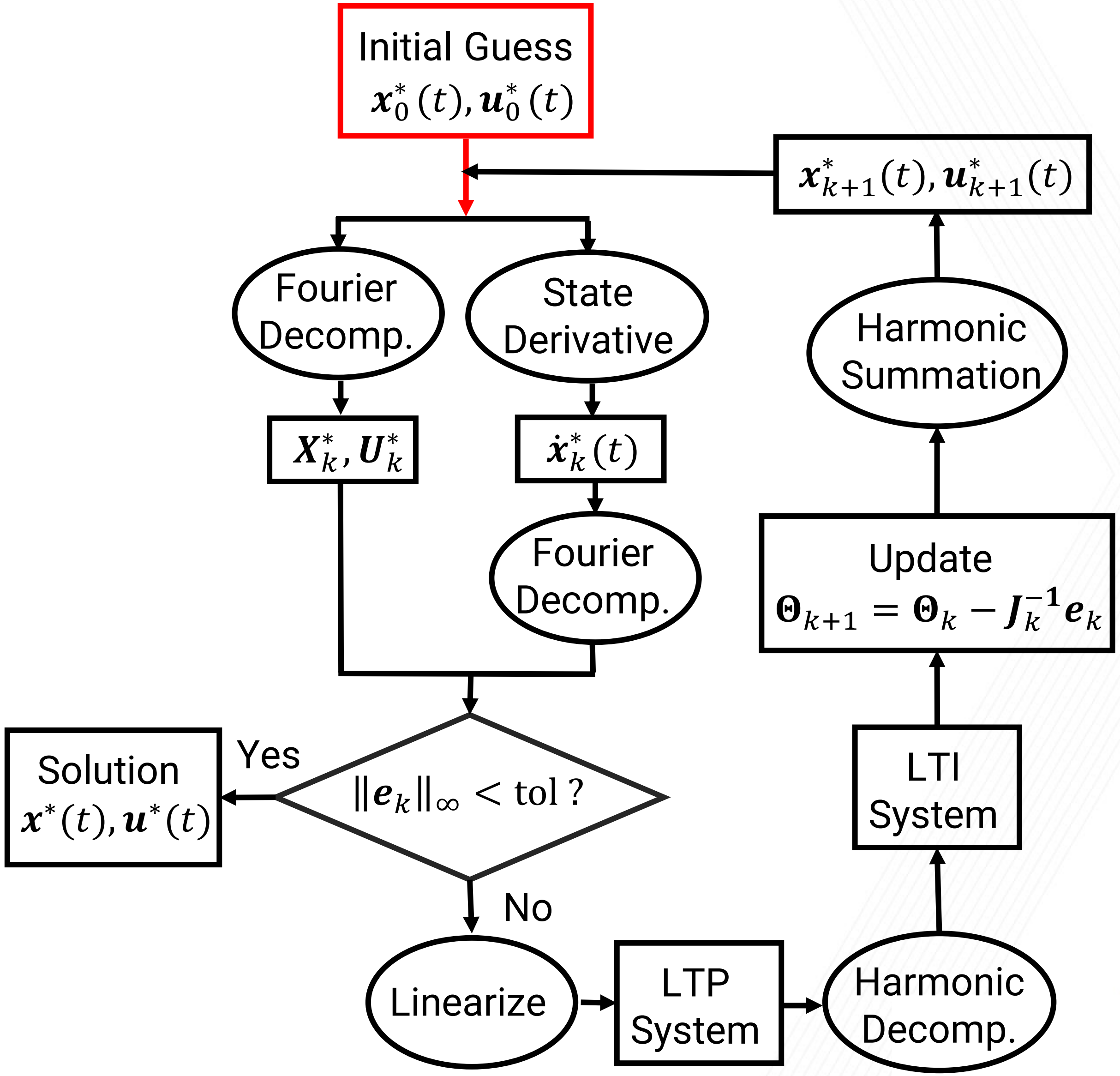
$$\dot{\boldsymbol{x}}^* = \boldsymbol{f}(\boldsymbol{x}^*, \boldsymbol{u}^*, t)$$

  with $\dot{\boldsymbol{x}}^*(t) = \dot{\boldsymbol{x}}^*(t + T)$

Numerical Method

Step 1

- Assume fundamental period T is known
- Fundamental period related to forcing frequency via $T = \frac{2\pi}{\omega}$
- Iterative algorithm
  - ❑Need for initial guess
  - ❑Candidate solution is refined at each iteration
  - ❑Stopped when convergence criteria is met
- Choose initial guess $\boldsymbol{x}_0^*(t), \boldsymbol{u}_0^*(t)$



Numerical Method

Step 2

- Iteration begins with decomposing candidate periodic solution in finite number of harmonics

$$\boldsymbol{x}_k^* = \boldsymbol{x}_{k_0}^* + \sum_{i=1}^{N} \boldsymbol{x}_{k_{1c}}^* \cos\left(\frac{2\pi i t}{T}\right) + \boldsymbol{x}_{k_{1s}}^* \cos\left(\frac{2\pi i t}{T}\right)$$

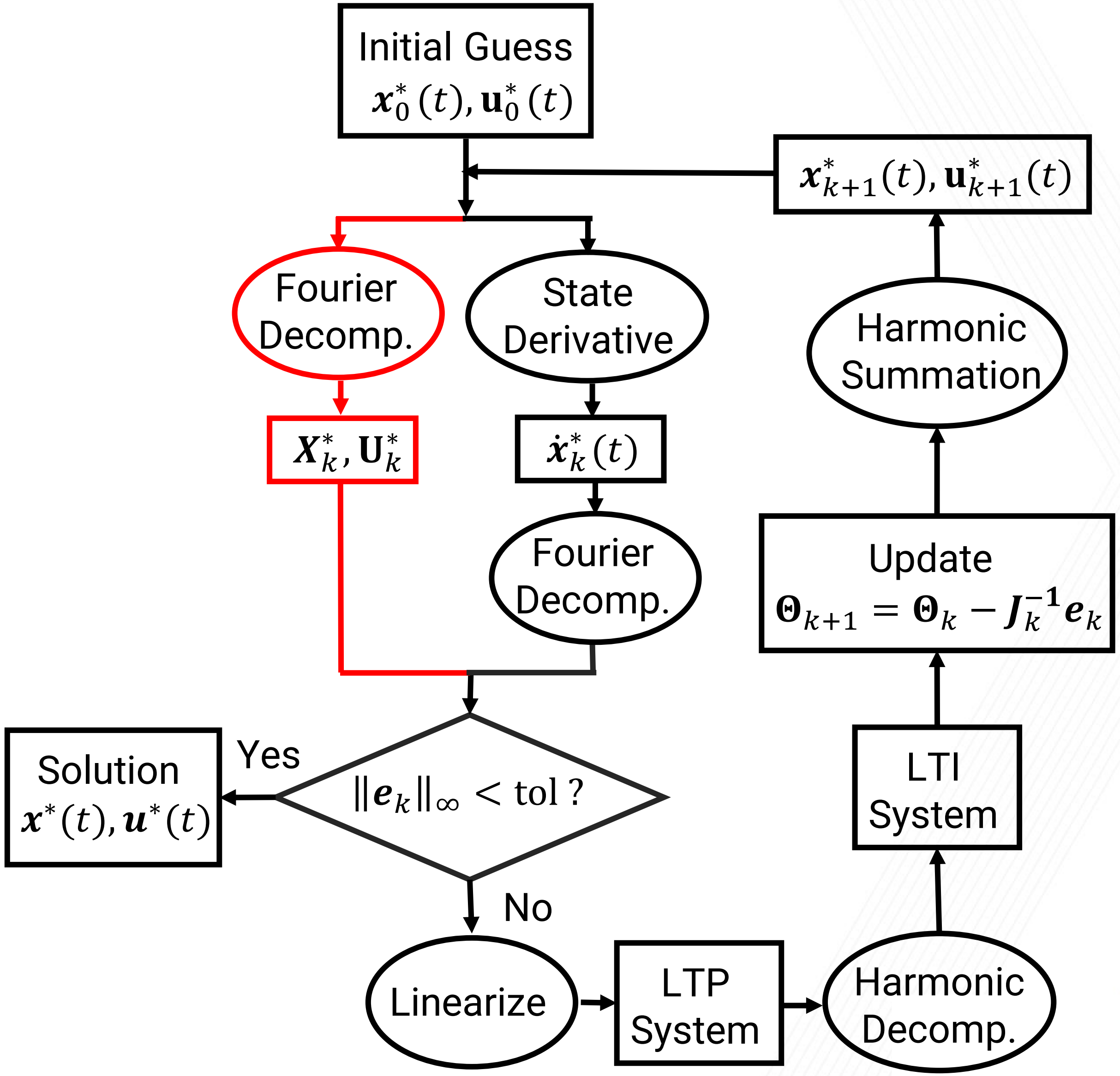
$$\boldsymbol{u}_k^* = \boldsymbol{u}_{k_0}^* + \sum_{j=1}^{M} \boldsymbol{u}_{k_{1c}}^* \cos\left(\frac{2\pi j t}{T}\right) + \boldsymbol{u}_{k_{1s}}^* \cos\left(\frac{2\pi j t}{T}\right)$$

- Re-write candidate solution in terms of Fourier coeffs.

$$\boldsymbol{X}_k^{*T} = \left[\boldsymbol{x}_{k_0}^* \boldsymbol{x}_{k_{1c}}^* \boldsymbol{x}_{k_{1s}}^* \dots \boldsymbol{x}_{k_{Nc}}^* \boldsymbol{x}_{k_{Ns}}^*\right]$$
$$\boldsymbol{U}_k^{*T} = \left[\boldsymbol{u}_{k_0}^* \boldsymbol{u}_{k_{1c}}^* \boldsymbol{u}_{k_{1s}}^* \dots \boldsymbol{u}_{k_{Mc}}^* \boldsymbol{u}_{k_{Ms}}^*\right]$$

- Form vector of unknowns at iteration k

$$\boldsymbol{\Theta}_k^T = \left[\boldsymbol{X}_k^{*T} \; \boldsymbol{U}_k^{*T}\right] \in \mathbb{R}^{n(2N+1)+m(2M+1)}$$

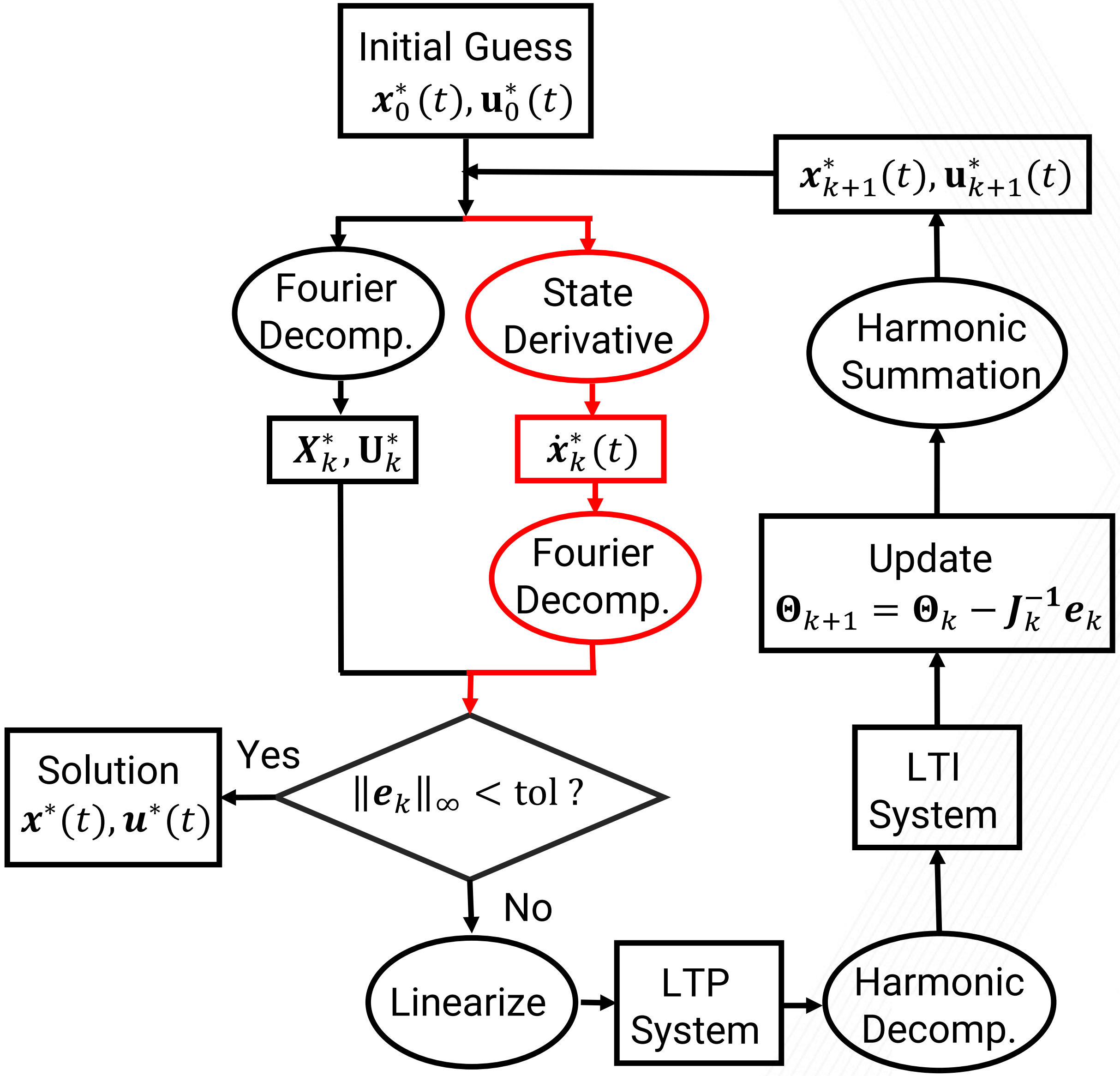


Numerical Method

Step 2a

- Calculate state derivative vector $\dot{\boldsymbol{x}}_k^*(t)$ along candidate periodic solution
- Decompose into N harmonics

$$\dot{\boldsymbol{x}}_k^* = \dot{\boldsymbol{x}}_{k_0}^* + \sum_{i=1}^{N} \dot{\boldsymbol{x}}_{k_{1c}}^* \cos\left(\frac{2\pi i t}{T}\right) + \dot{\boldsymbol{x}}_{k_{1s}}^* \cos\left(\frac{2\pi i t}{T}\right)$$



Numerical Method

Step 3

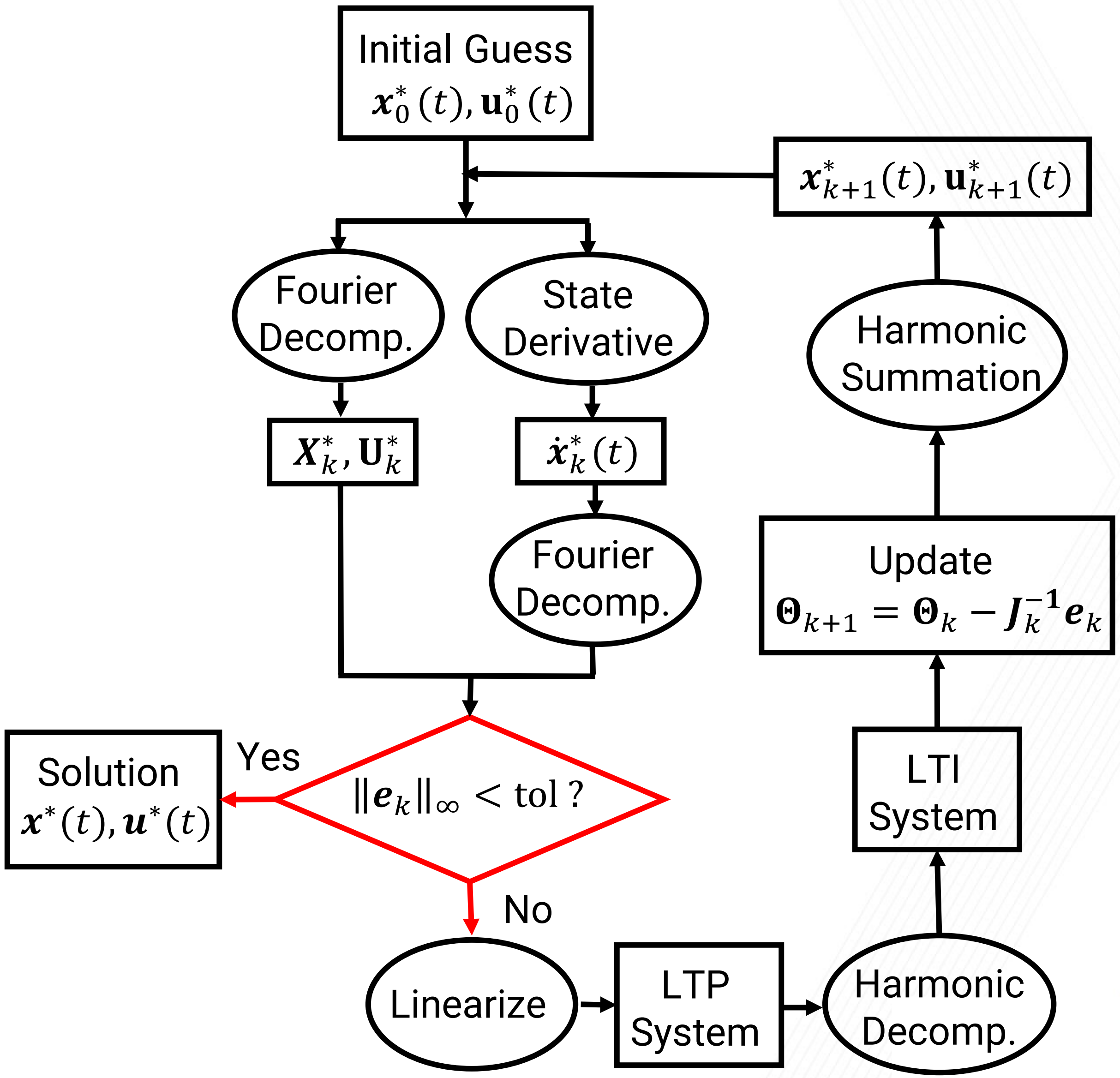
- Define error vector based on integral relations

$$\boldsymbol{e}_k^T = \boldsymbol{W}\left[(\dot{\boldsymbol{x}}_k^*)^T \left(\dot{\boldsymbol{x}}_{k_{1c}}^* - \frac{2\pi i}{T}\boldsymbol{x}_{k_{1s}}^*\right)^T \left(\dot{\boldsymbol{x}}_{k_{1s}}^* + \frac{2\pi i}{T}\boldsymbol{x}_{k_{1c}}^*\right)^T\right]$$

where

- ❑ $\boldsymbol{e}_k^T \in \mathbb{R}^{n(2N+1)}$
- ❑ $\boldsymbol{W} \in \mathbb{R}^{n(2N+1)\times n(2N+1)}$ diagonal scaling matrix

- If $\|\boldsymbol{e}_k\|_\infty$ is less than arbitrary tolerance, then solution is found
- If not, algorithm proceeds



Numerical Method

Step 4

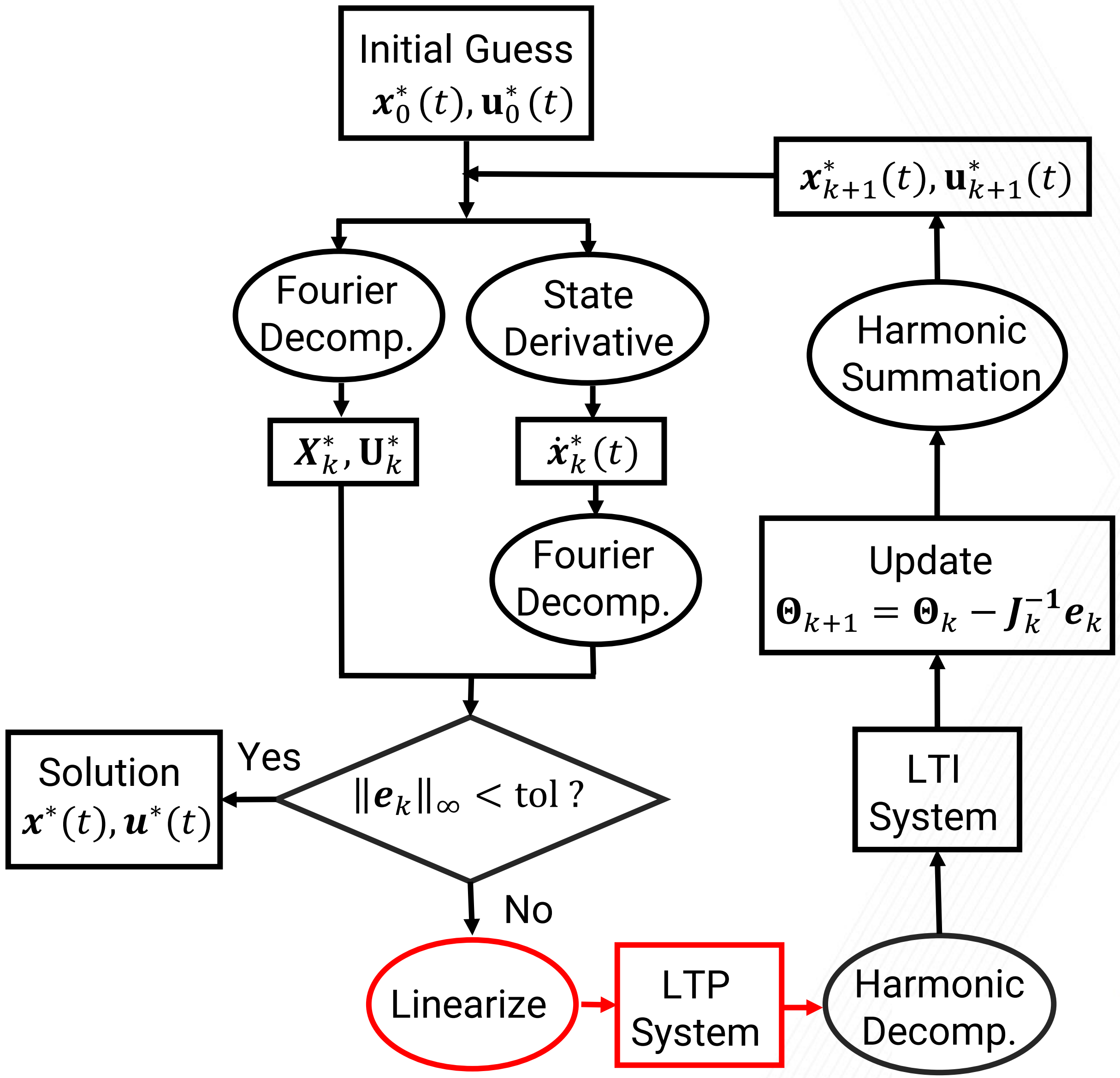
- NLTP system linearized about candidate periodic orbit to yield LTP system

$$\Delta\dot{x} = F_k(t)\Delta x + G_k(t)\Delta u$$

where

❑ $F_k(t) = \frac{\partial f(x,u,t)}{\partial x}\Big|_{x_k^*,\, u_k^*}$

❑ $G_k(t) = \frac{\partial f(x,u,t)}{\partial u}\Big|_{x_k^*,\, u_k^*}$



Numerical Method

Step 5

- Approximate LTP system with higher-order LTI model via harmonic decomposition

$$\Delta\dot{X} = A_k \Delta X + B_k \Delta U$$

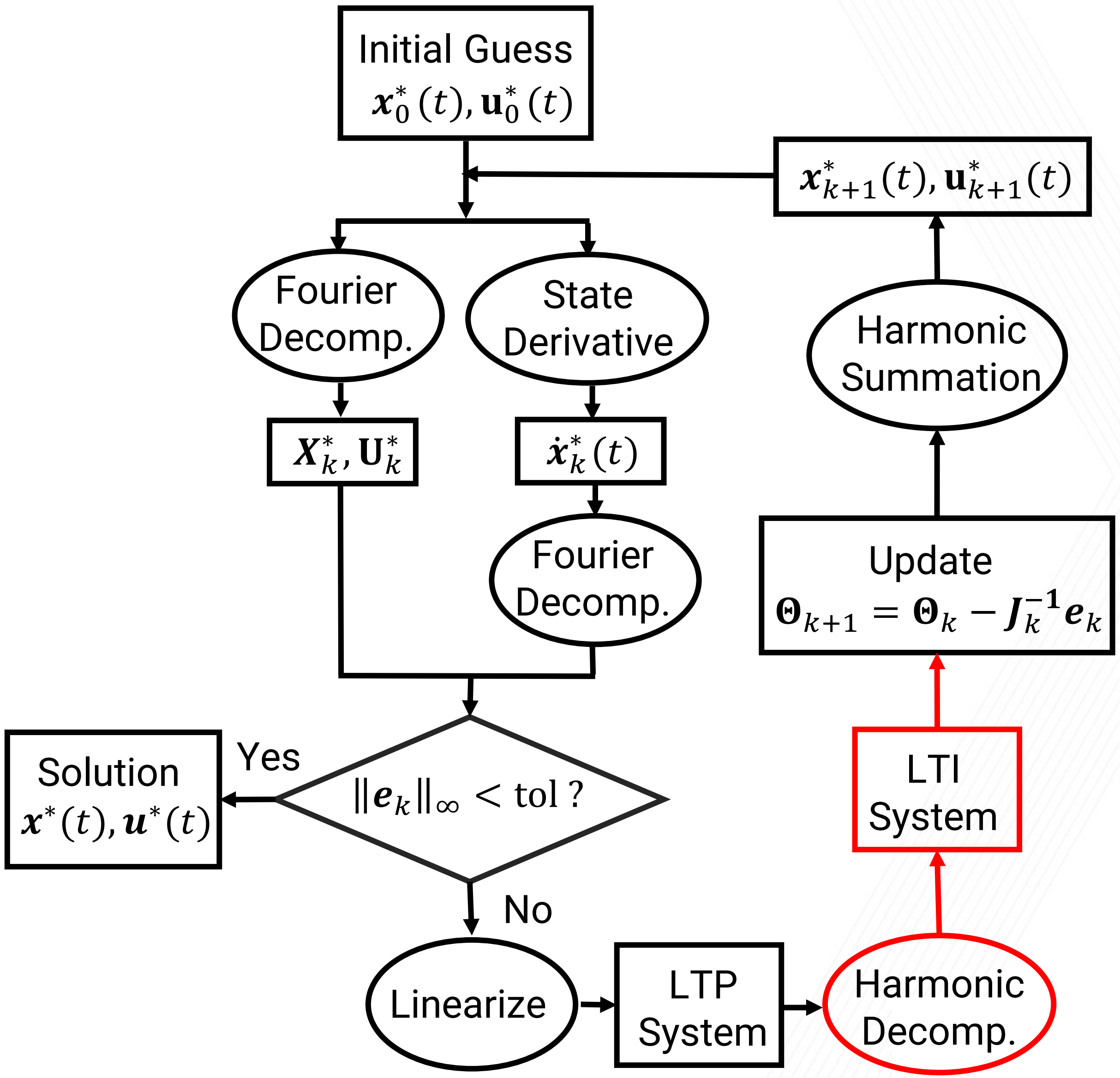
where

❑ $\Delta X^T = [\Delta x_0^T \ \Delta x_{1c}^T \ \Delta x_{1s}^T \ldots \Delta x_{Nc}^T \ \Delta x_{Ns}^T]$

❑ $\Delta U^T = [\Delta u_0^T \ \Delta u_{1c}^T \ \Delta u_{1s}^T \ldots \Delta u_{Mc}^T \ \Delta u_{Ms}^T]$

❑ $A_k \in \mathbb{R}^{n(2N+1)\times n(2N+1)}$

❑ $B_k \in \mathbb{R}^{n(2N+1)\times m(2M+1)}$



Numerical Method

Step 6

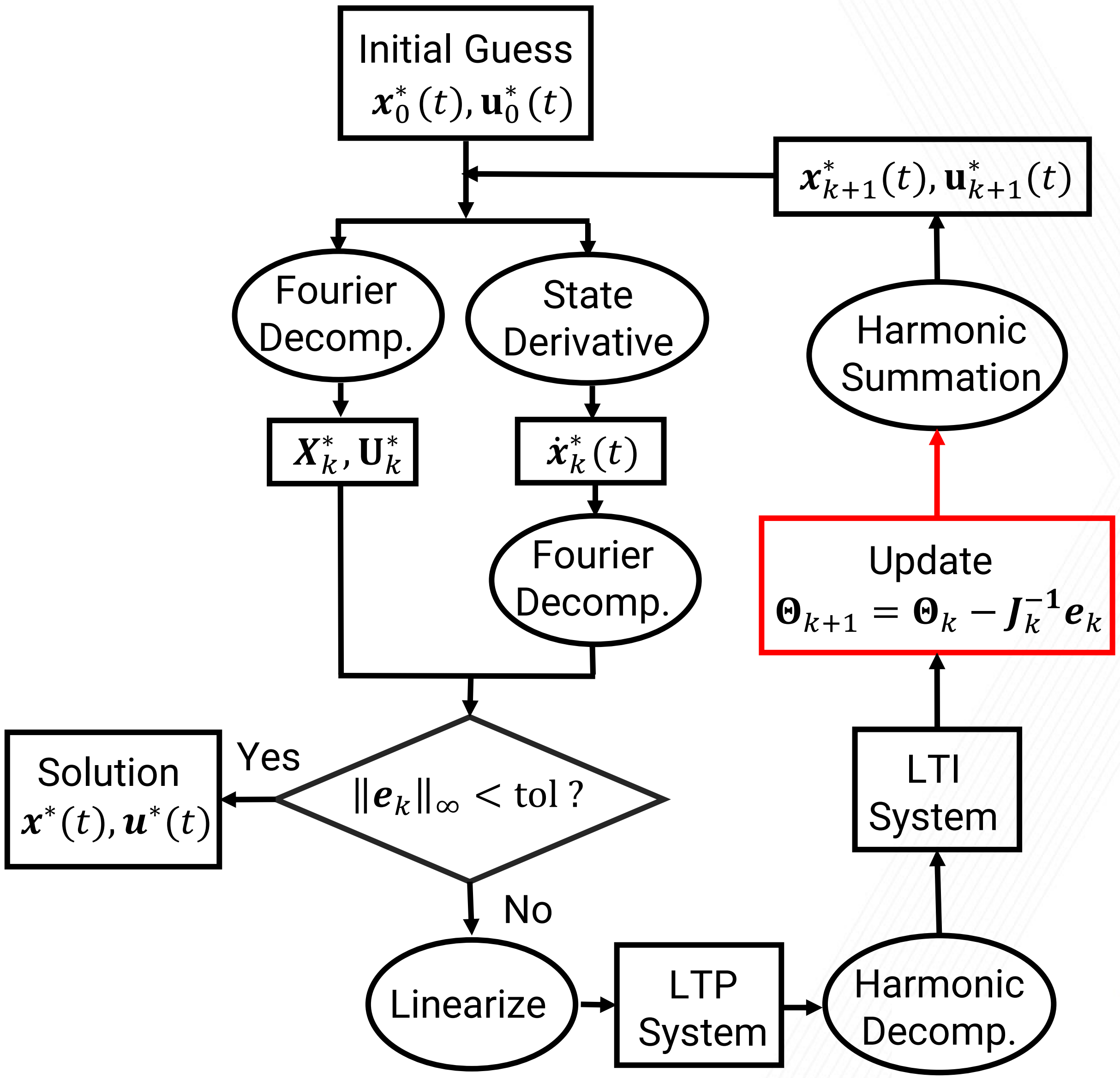
- Define the Jacobian matrix of the harmonic balancing algorithm at iteration k

$$\boldsymbol{J}_k = [\boldsymbol{A}_k \ \boldsymbol{B}_k]$$

  where

  - ❑ $\boldsymbol{J}_k \in \mathbb{R}^{n(2N+1)\times[n(2N+1)+m(2M+1)]}$

- Underdetermined problem with no unique solution
  - ❑ $n(2N+1) + m(2M+1)$ **unknowns**
  - ❑ $n(2N+1)$ **constraints**
- Need to specify $m(2M+1)$ trim conditions

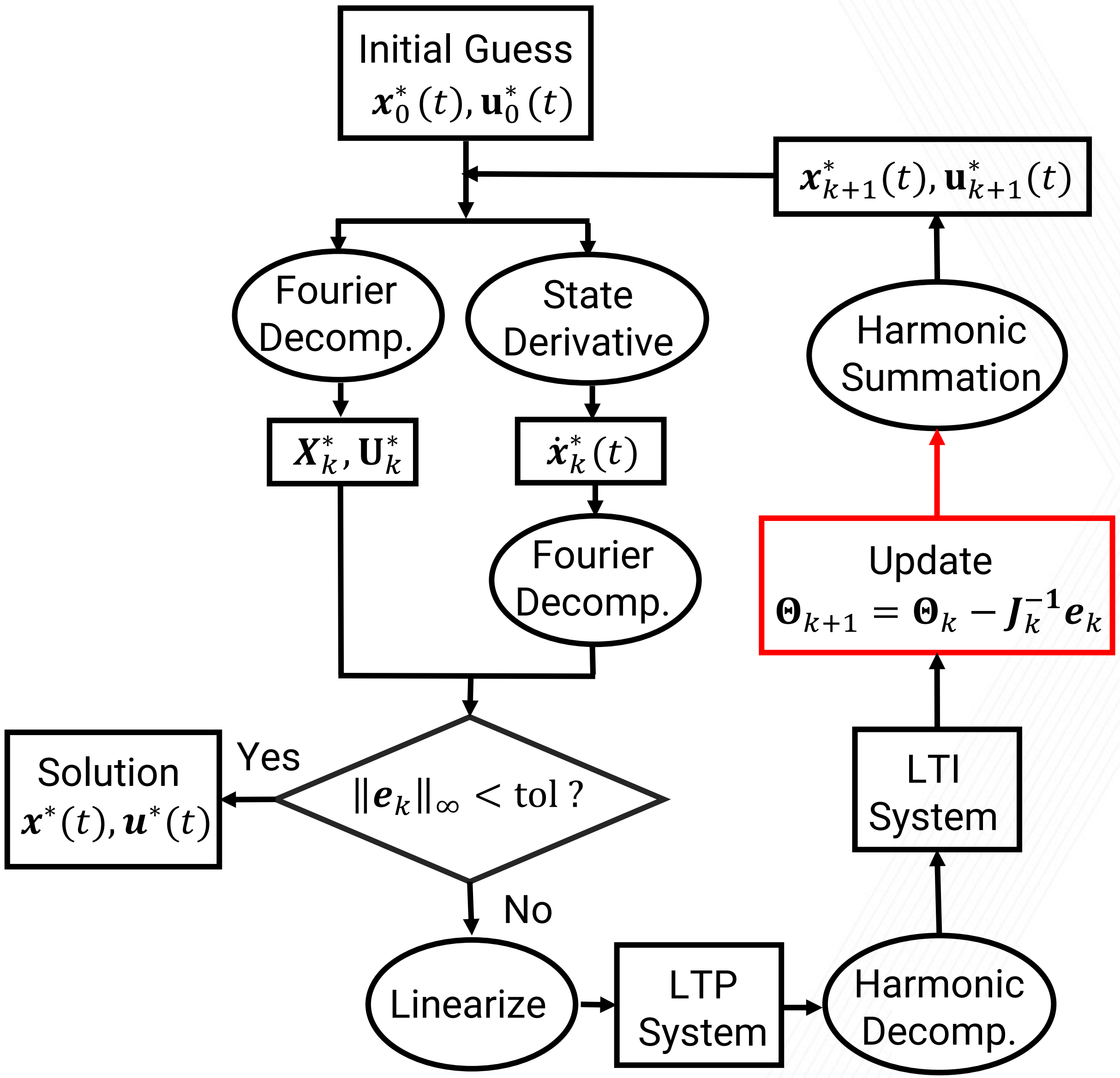


Numerical Method

Step 6 (cont'd)

- Can assume control input harmonics higher than 0th to be zero for typical aerospace vehicles
- This corresponds to imposing $2Mm$ conditions
- Remaining m conditions specified by fixing position (x, y, z) and heading (ψ)
  - ❑ Vehicles typically employ 4 control inputs
  - ❑ Dynamics invariant wrt position and heading
- Newton-Rhapson used for update

$$\boldsymbol{\Theta}_{k+1} = \boldsymbol{\Theta}_k - \boldsymbol{J}_k^{-1}\boldsymbol{e}_k$$



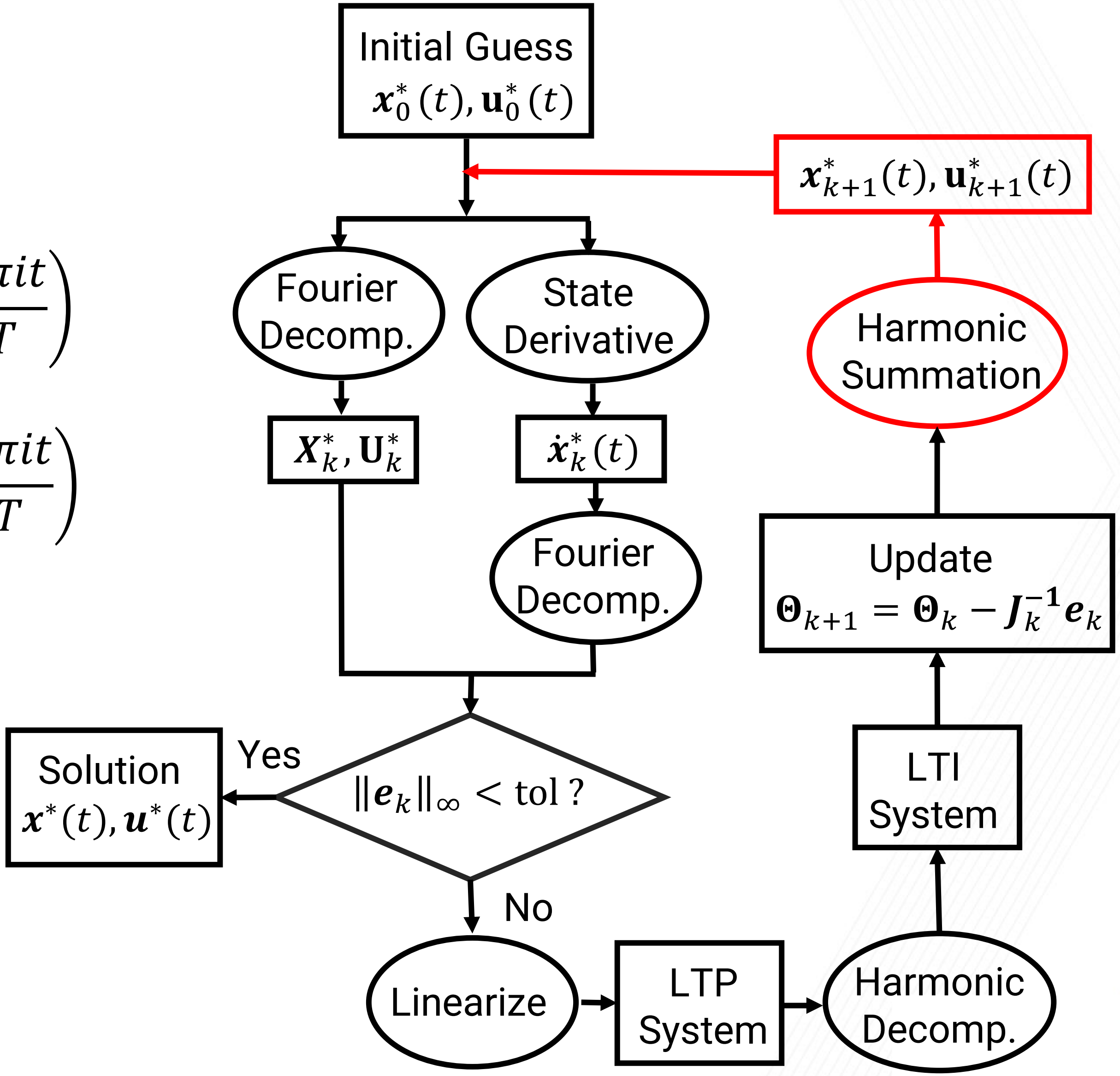
Numerical Method

Step 7

- New candidate periodic solution reconstructed in time domain

$$\boldsymbol{x}^*_{k+1} = \boldsymbol{x}^*_{k+1_0} + \sum_{i=1}^{N} \boldsymbol{x}^*_{k+1_{1c}} \cos\left(\frac{2\pi i t}{T}\right) + \boldsymbol{x}^*_{k+1_{1s}} \cos\left(\frac{2\pi i t}{T}\right)$$

$$\boldsymbol{u}^*_{k+1} = \boldsymbol{u}^*_{k+1_0} + \sum_{i=1}^{M} \boldsymbol{u}^*_{k+1_{1c}} \cos\left(\frac{2\pi i t}{T}\right) + \boldsymbol{u}^*_{k+1_{1s}} \cos\left(\frac{2\pi i t}{T}\right)$$



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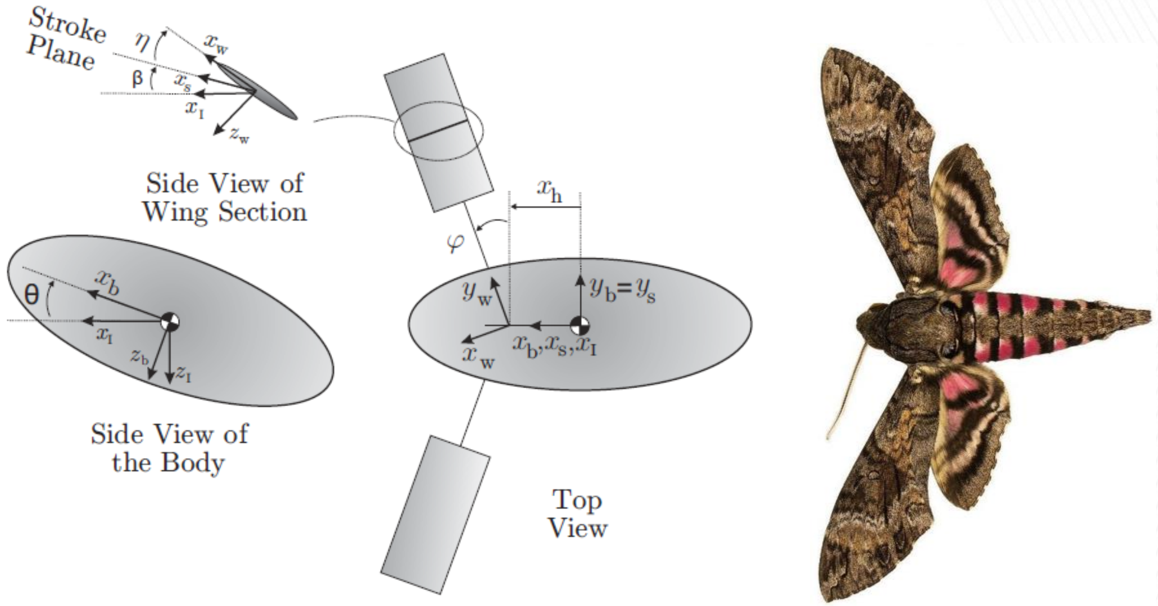
Flapping-Wing MAV

Vertical Dynamics Model [Hassan et al. 2016]

- Flapping-wing MAV representative of a **hawk moth**
- NLTP vertical dynamics

$$\begin{bmatrix} \dot{z} \\ \dot{\phi} \\ \dot{w} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} w \\ \dot{\phi} \\ g - k_{d_1}|\dot{\phi}|w - k_L\dot{\phi}^2 \\ -k_{d_2}|\dot{\phi}|\dot{\phi} - k_{d_3}w\dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{I_F}\cos(\omega t) \end{bmatrix} U$$

- 4 states + 1 control input
  - ❑States: $\boldsymbol{x}^T = \{z\ \phi\ w\ \dot{\phi}\}$ $(n = 2)$
  - ❑Controls: $\boldsymbol{u} = U$ $(\mathrm{m} = 1)$
- Forcing frequency $\omega = 165$ rad/s

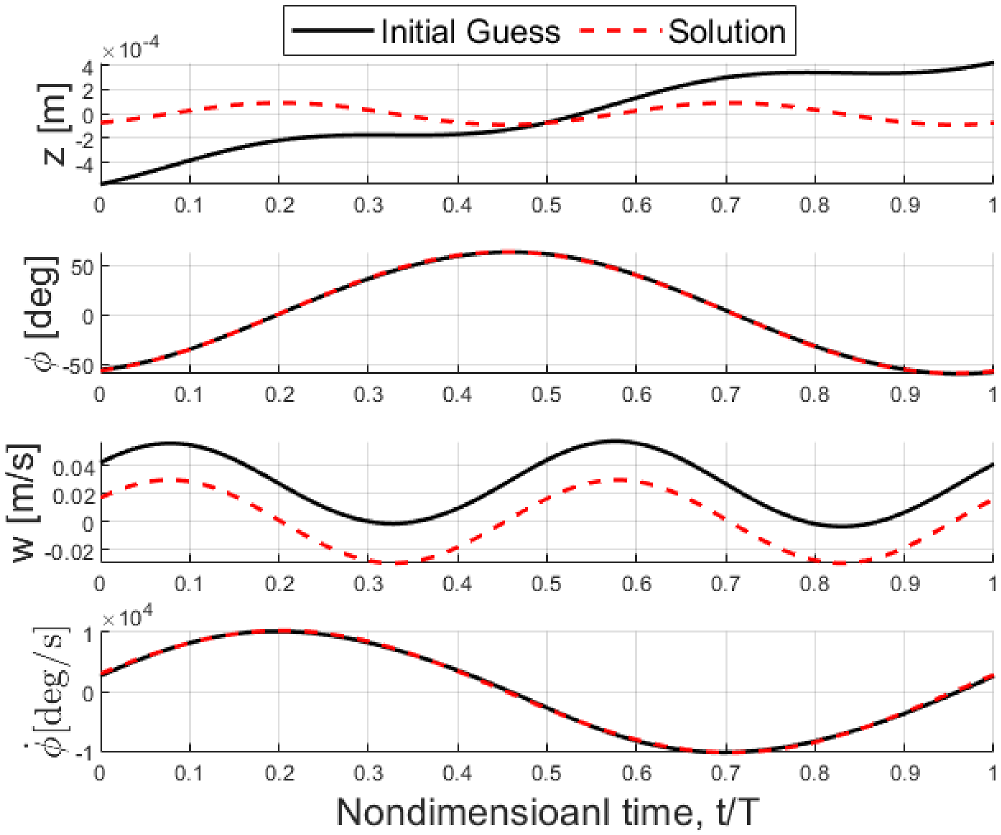


Schematic diagram of a FWMAV

Flapping-Wing MAV

Trim Solution

- **Objective**: find periodic trim in hover
- Harmonics retained
  - ❑Up to 2nd state harmonic ($N = 2$)
  - ❑0th control harmonic ($M = 0$)
- 21 unknowns vs. 20 constraints → set 0th harmonic of vertical position to zero
- Initial guess far from periodic orbit results in increasing number of iterations
- Computation time: 0.51 sec/iteration
- Higher-order LTI overlaps NLTP response to control input doublet



Periodic trim solution

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Flapping-Wing MAV

Trim Solution

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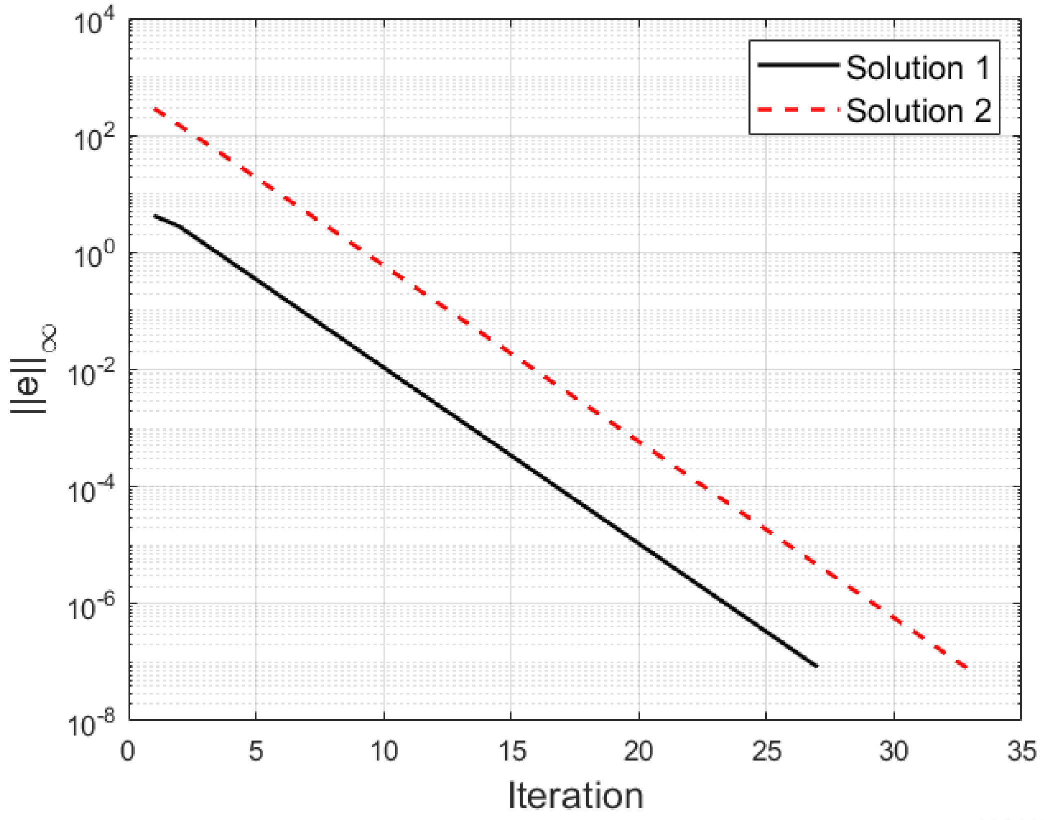

Convergence Properties

Flapping-Wing MAV

Trim Solution

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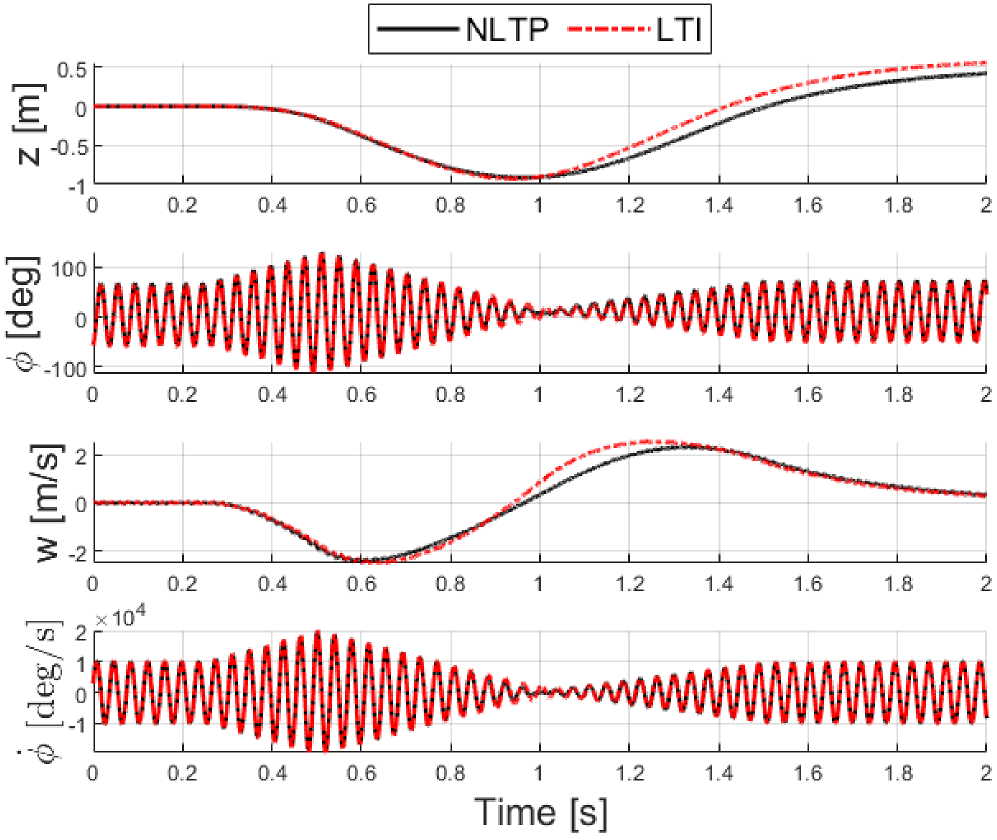


Doublet response: NLTP vs LTI

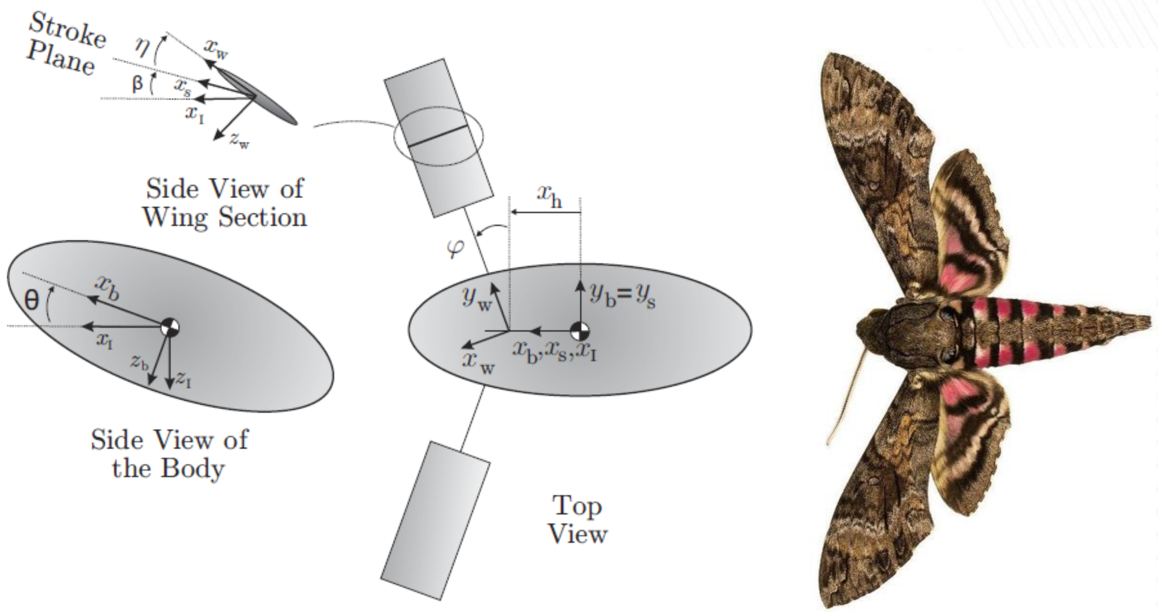
Flapping-Wing MAV

Longitudinal Dynamics Model [Taha 2014]

- Flapping-wing MAV representative of a **hawk moth**
- NLTP longitudinal dynamics

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} u\cos\theta + w\sin\theta \\ -u\sin\theta + w\cos\theta \\ -qw - g\sin\theta \\ qu + g\cos\theta \\ 0 \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ X(\boldsymbol{x},\boldsymbol{u},t)/m \\ Y(\boldsymbol{x},\boldsymbol{u},t)/m \\ M(\boldsymbol{x},\boldsymbol{u},t)/I_y \\ 0 \end{bmatrix}$$

- 6 states + 2 control inputs
  - ❑States: $\boldsymbol{x}^T = \{x\ z\ u\ w\ q\ \theta\}$
  - ❑Controls: $\boldsymbol{u}^T = \{\Phi\ \alpha_m\}$
- Forcing frequency $\omega = 165$ rad/s

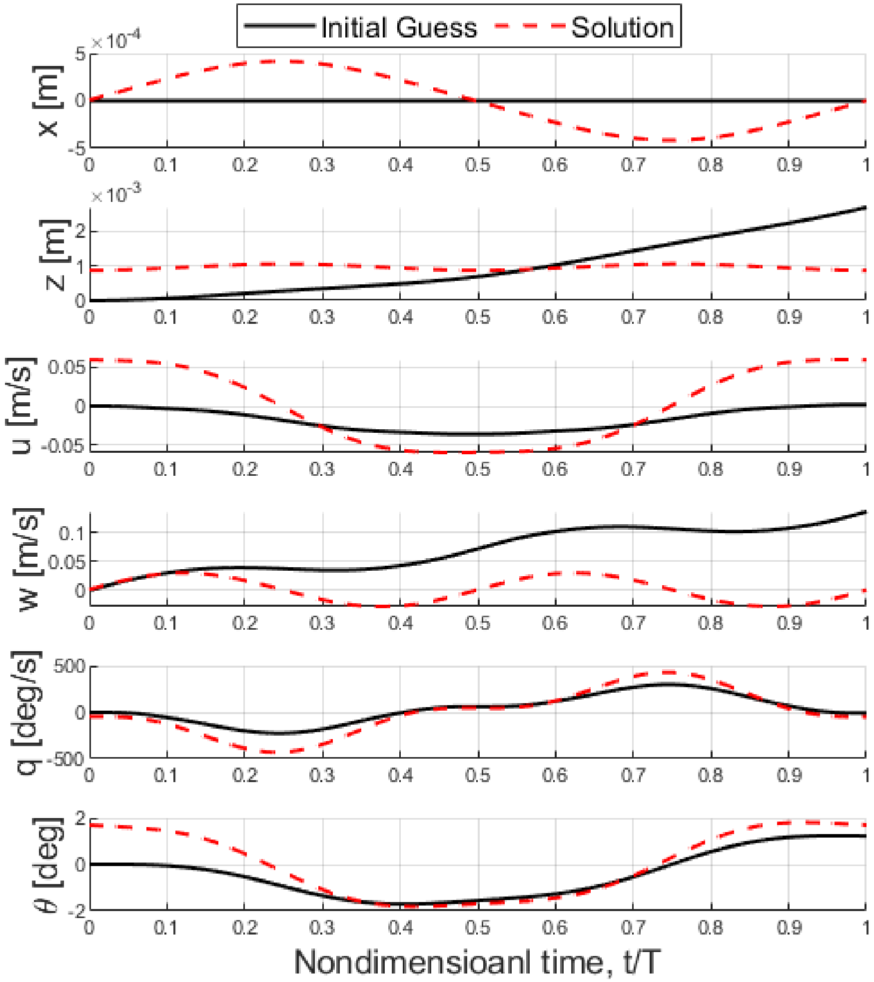


Schematic diagram of a FWMAV

Flapping-Wing MAV

Trim Solution

- **Objective**: find periodic trim in hover
- Harmonics retained
    - ❑Up to 4th state harmonic ($N = 4$)
    - ❑0th control harmonic ($M = 0$)
- 54 unknowns vs. 52 constraints → 0th harmonic of long. and vertical position set to zero
- Higher-order LTI overlaps NLTP response to control input doublet



Periodic Trim Solution

Flapping-Wing MAV

Trim Solution

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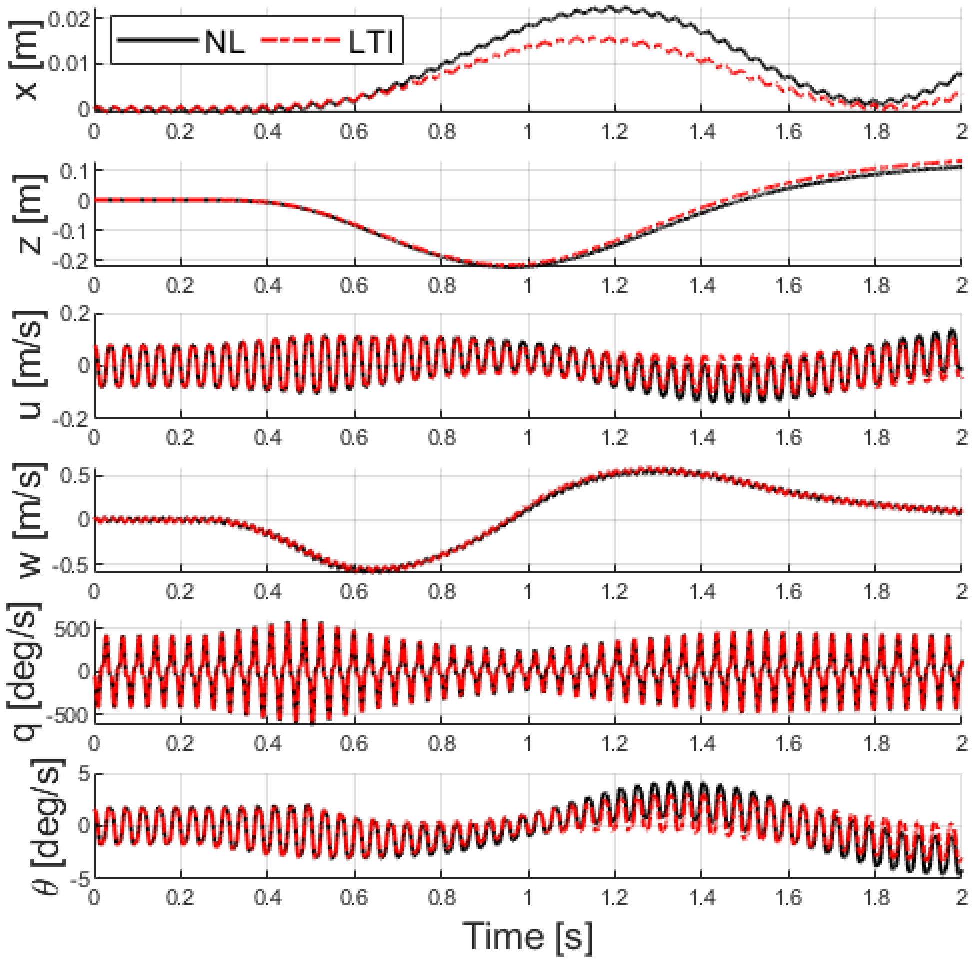

Doublet response: NLTP vs LTI

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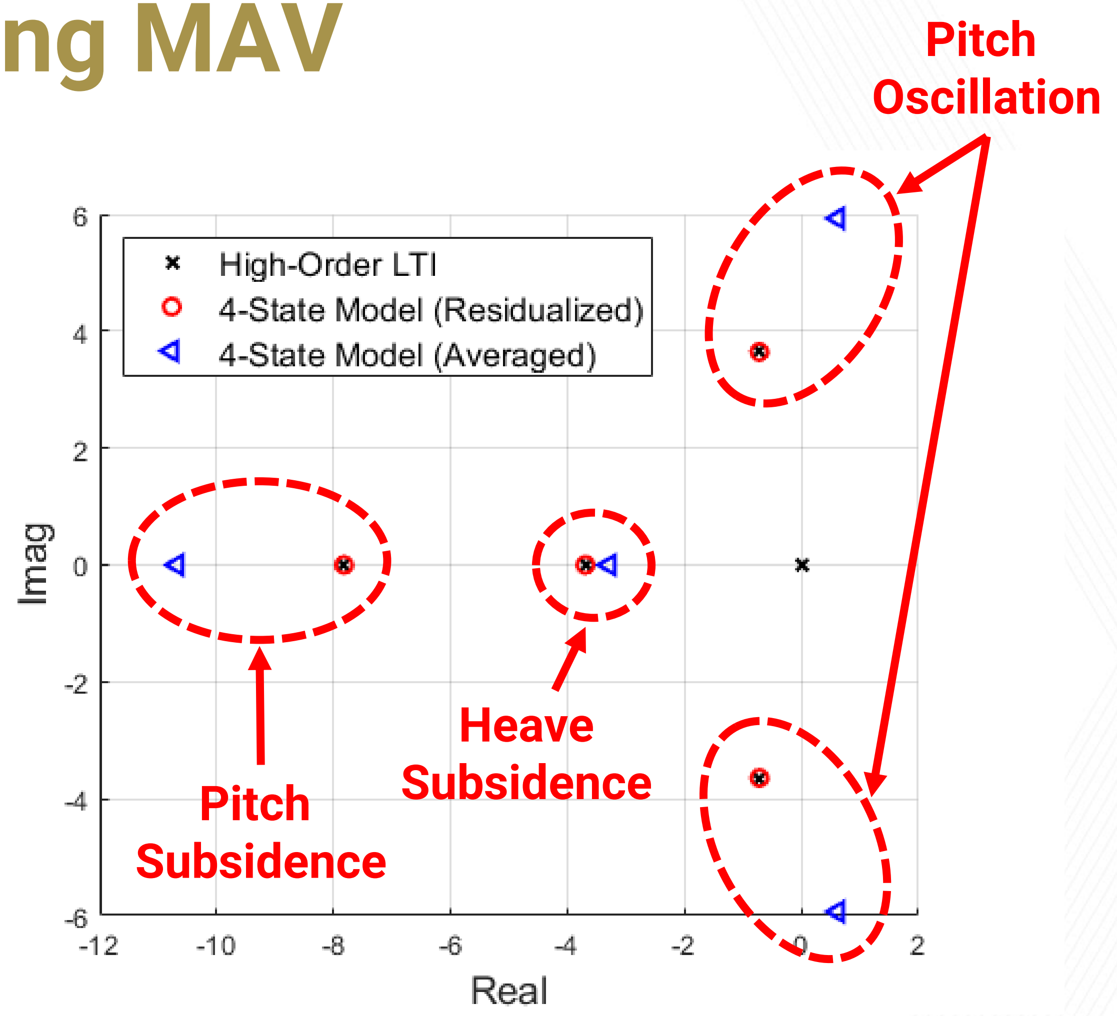
Flapping-Wing MAV

Spectral Analysis

- Higher-order LTI model is residualized to yield a 4-state model
- Low-frequency eigenvalues match those of the higher-order LTI
- Can clearly see hovering cubic
- Averaged dynamics indicate instability…
- **… but dynamics are stable!**
- System gains pitch stiffness as result of high-frequency, high-amplitude forcing



Eigenvalues in hover

Helicopter

Simulation Model

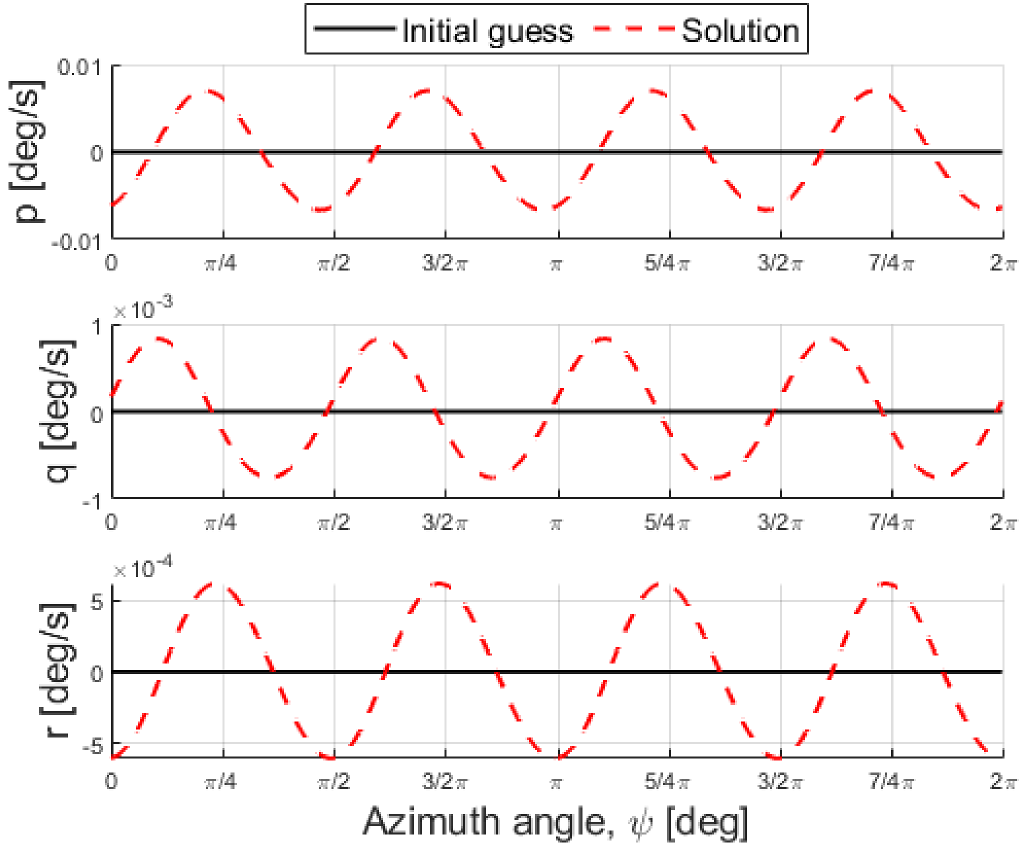
- Utility helicopter representative of UH-60
- julia implementation of **GenHel**
- Rigid flap + lead-lag
- Dynamic inflow model
  - ❑3-state Pitt-Peters (main rotor)
  - ❑1-state Bailey (tail rotor)
- Nonlinear aerodynamics
  - ❑Airframe
  - ❑Rotor blades
- 32 states + 4 control inputs
  - ❑Fuselage: $\boldsymbol{x}_F^T = \{u\ v\ w\ p\ q\ r\ \phi\ \theta\ \psi\ x\ y\ z\}$
  - ❑Rotor: $\boldsymbol{x}_R^T = \{\boldsymbol{\beta}_{\text{MBC}}^T\ \dot{\boldsymbol{\beta}}_{\text{MBC}}^T\ \boldsymbol{\zeta}_{\text{MBC}}^T\ \dot{\boldsymbol{\zeta}}_{\text{MBC}}^T\ \boldsymbol{\lambda}_{\text{MBC}}^T\ \lambda_{0T}\}$
  - ❑Controls: $\boldsymbol{u}^T = \{\delta_{\text{lat}}\ \delta_{\text{lon}}\ \delta_{\text{col}}\ \delta_{\text{ped}}\}$

UH-60 Black Hawk

Helicopter

Trim Solution

- **Objective**: find periodic trim at 120 kts
- Harmonics retained
  - ❑Up to 4th state harmonic ($N = 4$)
  - ❑0th control harmonic ($M = 0$)
- 288 unknowns vs. 284 constraints → set 0th harmonic of position and heading to zero
- Good agreement with flight test data
- Robust performance to poor initial guess
- Computation time: 40 sec/iteration

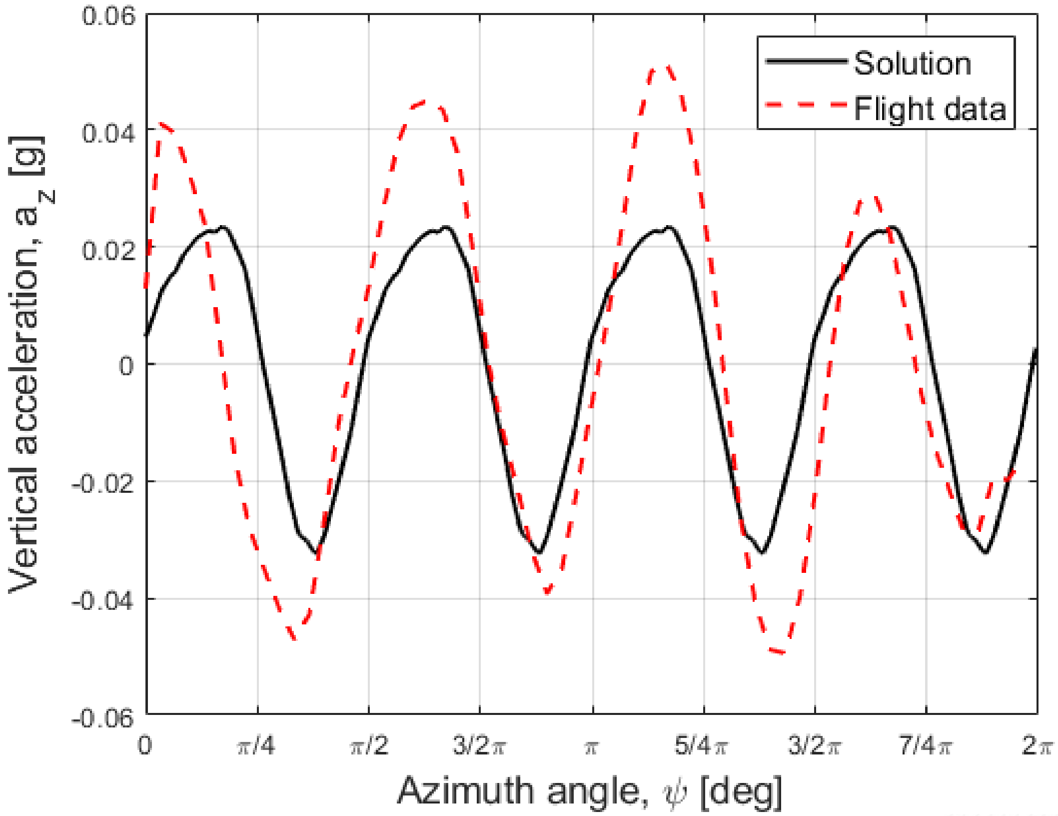


Periodic equilibrium

Helicopter

Trim Solution

- **Objective**: find periodic trim at 120 kts
- Harmonics retained
  - ❑Up to 4th state harmonic ($N = 4$)
  - ❑0th control harmonic ($M = 0$)
- 288 unknowns vs. 284 constraints → set 0th harmonic of position and heading to zero
- Good agreement with flight test data
- Robust performance to poor initial guess
- Computation time: 40 sec/iteration



Vertical acceleration over one rotor revolution: numerical solution vs. JUH-60A RASCAL flight data

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Helicopter

Trim Solution

- **Objective**: find periodic trim at 120 kts
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  - ❑Up to 4th state harmonic ($N = 4$)
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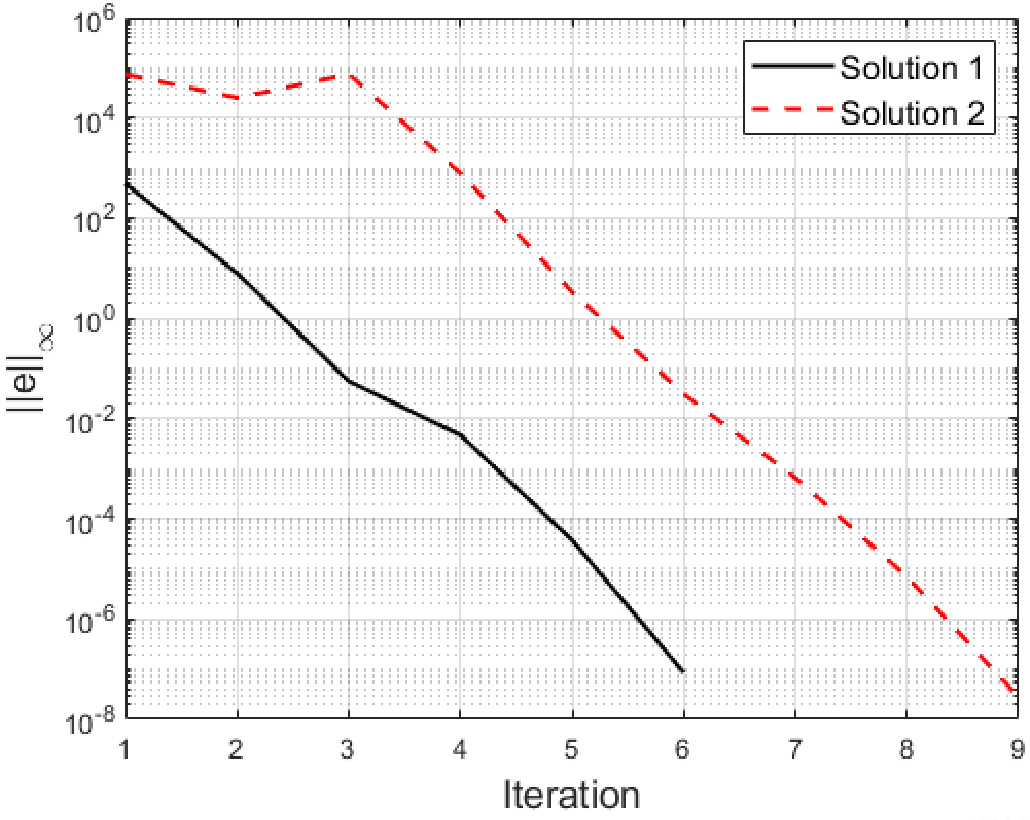

Convergence Properties

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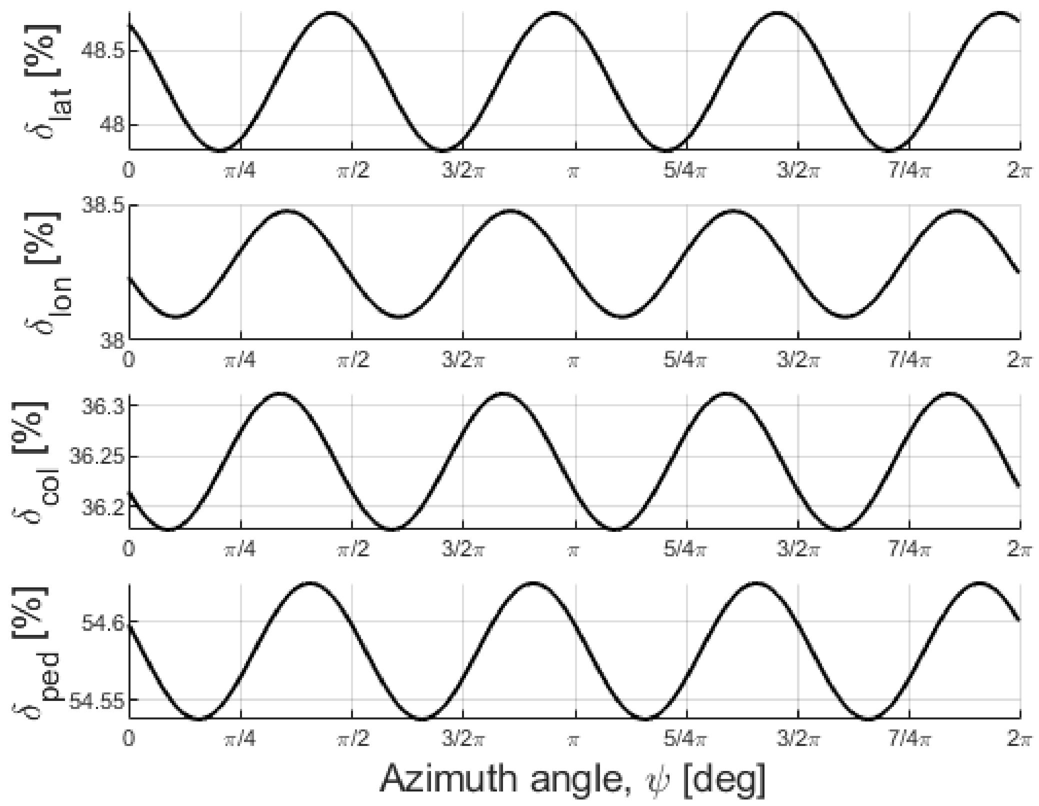
Helicopter

Trim Solution

- **Objective**: find higher-harmonic control (HHC) input to eliminate vibrations at CG at 120 kts
- Harmonics retained
  - ❑Up to 4th state harmonic ($N = 4$)
  - ❑**Up to 4th control harmonic** ($M = 4$)
- 324 unknowns vs. 288 constraints → all position and heading harmonics set to zero
- HHC input shows 4/rev behavior as expected



HHC input to eliminate vibrations at CG

Flapping-Tail Airplane

Simulation Model

- Flapping-tail concept airplane representative of a Boeing 737-800
- Horizontail tail subjected to oscillatory motion
  - ❑**Pitching** $\alpha(t) = A_\alpha \sin(\omega t + \phi_\alpha)$
  - ❑**Plunging** $h(t) = A_h \sin(\omega t + \phi_h)$
  - ❑**Flapping** $\delta(t) = A_\delta \sin(\omega t + \phi_\delta)$
- Longitudinal dynamics

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -qw - g\sin\theta \\ qu + g\cos\theta \\ 0 \\ q \\ u\cos\theta + w\sin\theta \end{bmatrix} + \begin{bmatrix} X(\boldsymbol{x},t)/m \\ Y(\boldsymbol{x},t)/m \\ M(\boldsymbol{x},t)/I_y \\ 0 \\ 0 \end{bmatrix}$$

- 5 states + 0 control inputs
  - ❑States: $\boldsymbol{x}^T = \{u\ w\ q\ \theta\ x\}$

Boeing 737-800

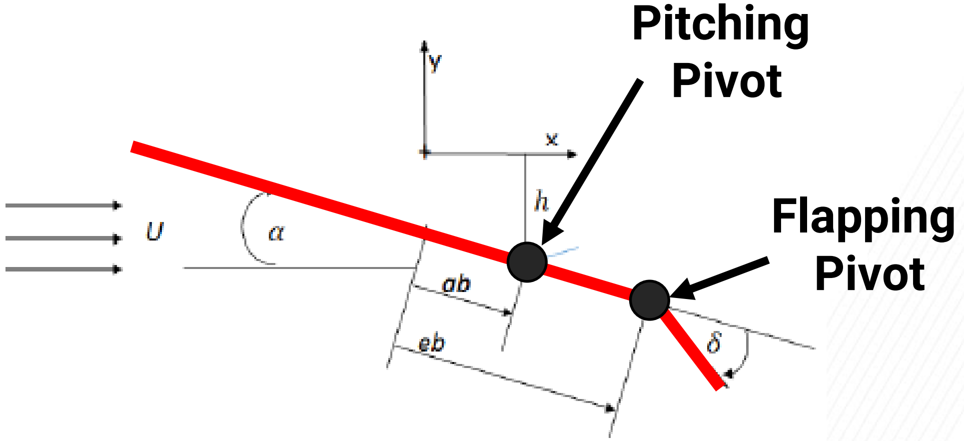


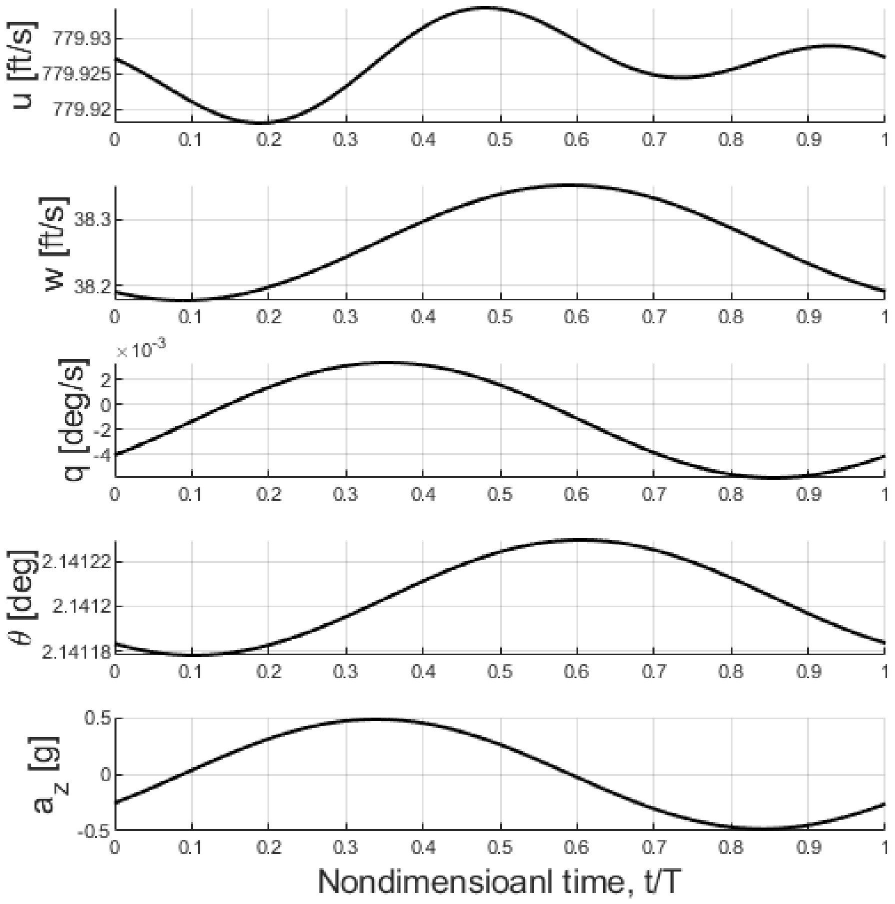
Diagram of oscillating tail

Flapping-Tail Airplane

Trim Solution

- **Objective**: find periodic trim at 780 ft/s
- Harmonics retained
  - ❑Up to 2nd state harmonic ($N = 2$)
- 25 unknowns vs. 25 constraints → OK!
- Vibration level prediction
  - ❑**Flapping tail plane**: 0.5 g
  - ❑**Typical airliners**: 0.01 g
  - ❑**Helicopters**: 0.05 - 0.1 g

Not Comfortable



Periodic trim solution

- **Introduction**
  - ❑Background
  - ❑Motivation
  - ❑Objectives
- **Methodology**
  - ❑Problem Definition
  - ❑Numerical Methods
- **Results**
  - ❑Flapping-Wing MAV
  - ❑Helicopter
  - ❑Flapping-Tail Airplane
- **Conclusions**
  - ❑Broader Implications
  - ❑Future Directions

Conclusions

1. **Developed a numerical method for determining the periodic state and control solutions of nonlinear time-periodic systems**
   - ❑Harmonic balance revisitation that employs high-order LTI models of the vehicle dynamics
   - ❑Based on harmonic decomposition
   - ❑Does not rely on state transition matrices
   - ❑Simultaneously solves for the approximate higher-order LTI dynamics
   - ❑Can be used to compute the high-harmonic control inputs that attenuate arbitrary state harmonics
2. **Algorithm applications**
   - ❑Development of advanced flight control laws that attenuate certain state harmonics
   - ❑Prediction of loads and vibrations

Broader Implications

NLTP

Trim Non-Linear Time-Periodic (NLTP) System

- Find states and controls that result in periodic orbit

LTP

Obtain Linear Time-Periodic (LTP) System

- Linearization schemes

LTI

Transform LTP system into higher-order Linear Time Invariant (LTI) system

- Harmonic Decomposition
- Stability Analysis

Reduced Order LTI

Model Order Reduction

- Reduce states that cannot be observed/measured

Control Design

Harmonic Control Design

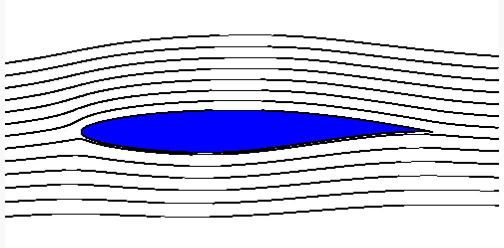
- Enhance stability/meet desired response characteristics
- Minimize arbitrary state/output harmonics

Future Directions

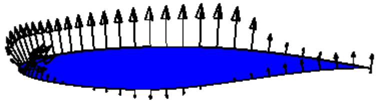
Linear Time-Invariant Models of Rotorcraft Flight Dynamics, Vibrations, and Acoustics

Ffowcs Williams-Hawkings Equation

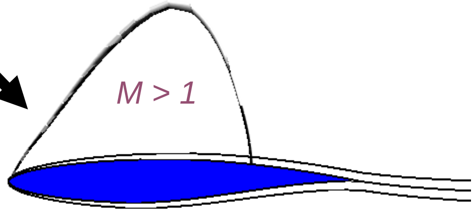
$$\nabla^2 p'(\boldsymbol{x}, t) = \frac{\partial}{\partial t}\left[Q\delta(f)\right] - \frac{\partial}{\partial x_i}\left[F_i\delta(f)\right] + \frac{\partial}{\partial x_i x_j}\left[T_{ij}H(f)\right]$$



Thickness
displacement of fluid generates sound



Loading
accelerating force distribution generates sound (includes **BVI noise**)



Quadrupole
All volume sources, non-linear effects nonuniform sound speed

Courtesy of K. S. Brentner

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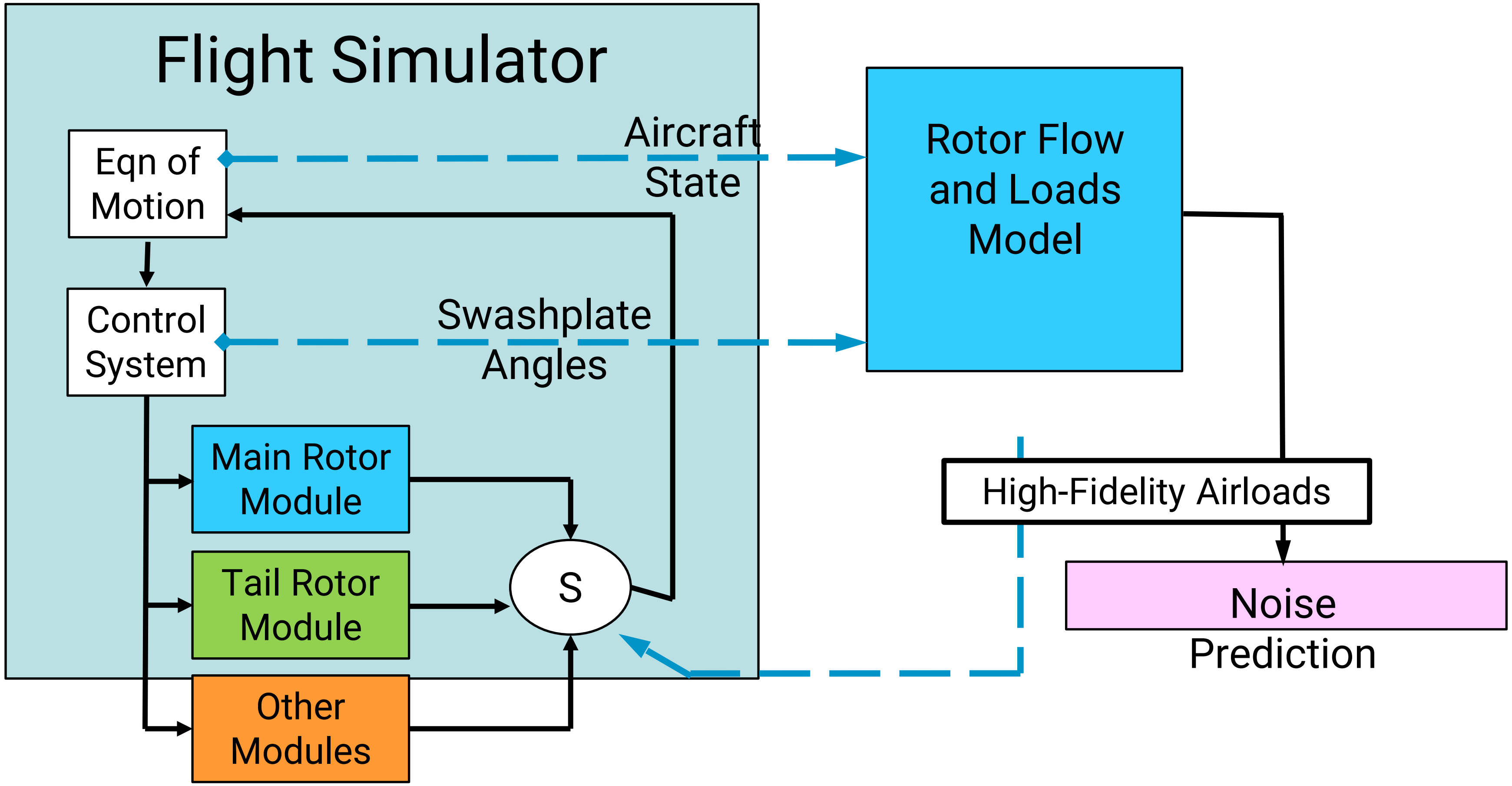
Future Directions

Linear Time-Invariant Models of Rotorcraft Flight Dynamics, Vibrations, and Acoustics



Courtesy of K. S. Brentner and Mrunali Botre

Future Directions

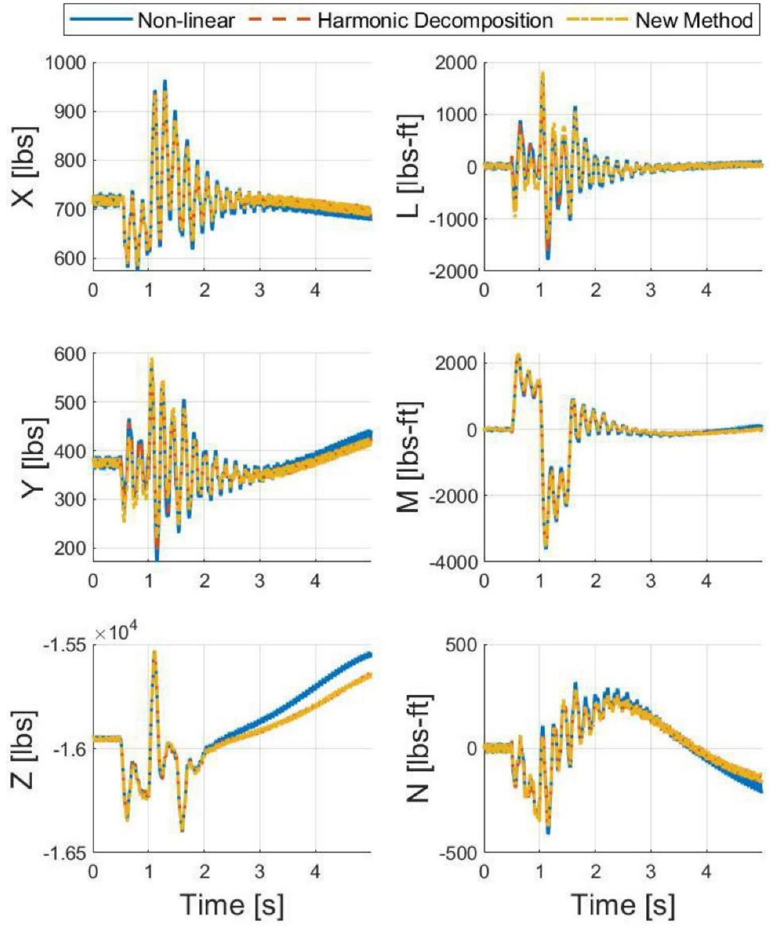
Linear Time-Invariant Models of Rotorcraft Flight Dynamics, Vibrations, and Acoustics

Objectives

- Include vibration and aeroacoustic as output of NLTP dynamics
- Derive high-order LTI models for use in vibrations and acoustic predictions
- Drastically abate computation time
- Develop noise-abating control laws based on LTI system theory → powerful
- **Noise on urban areas (UAM)**
- **Cabin noise**

Funding Agencies / Interactions

- VLRCOE (NASA): coordinated task with **K.S. Brentner** and **J.F. Horn**
- **E. Grennwood** (PSU)
- FAA Center of Excellence (ASCENT)
- SBIR/STTR (ART, CDI, Craft Tech)



Preliminary results from novel algorithm

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Future Directions

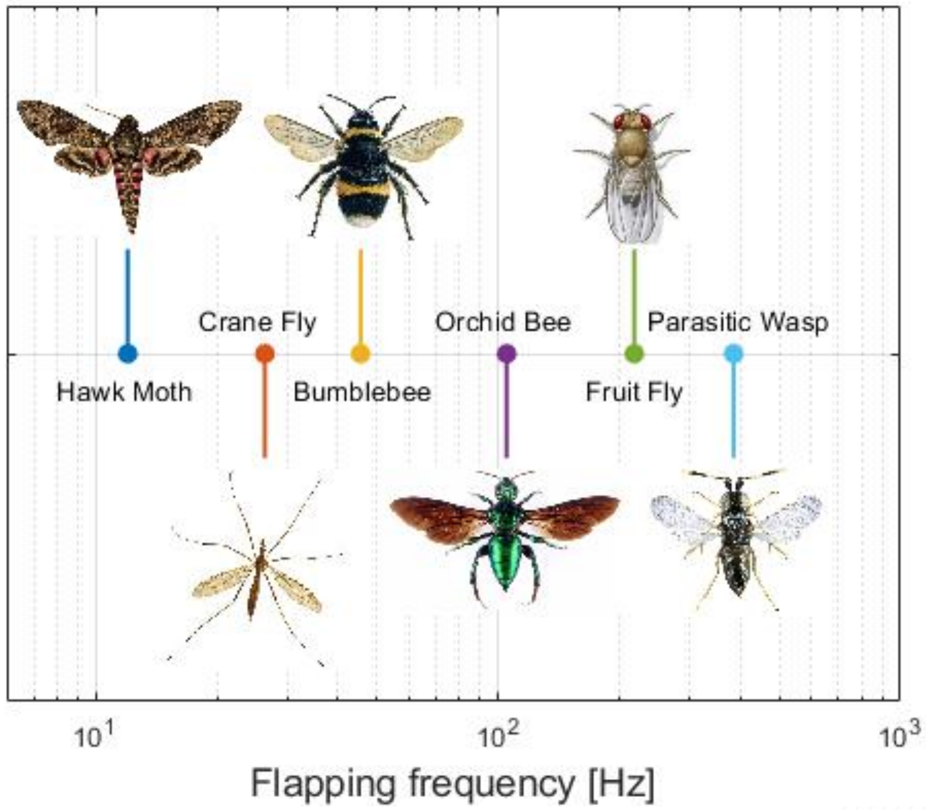
Stability and Control of Flapping-Wing Flight

Objectives

- Extend proposed method to more complex FWMAV models including unsteady aerodynamics states
- Assess stability of wide spectrum of bio flyers, especially those with small time scale separation
- Compare with averaging methods
- Perform control design based on high-order LTI systems that accounts for higher harmonics

Funding Agencies / Interactions

- ONR (**Marc Steinberg**)
- NSF DCSD
- **H. Taha** (UCI)



Flapping Frequency for Several Biological Flyers

Future Directions

Linear Time-Invariant Approximations of Spacecraft Formation Dynamics

Problem

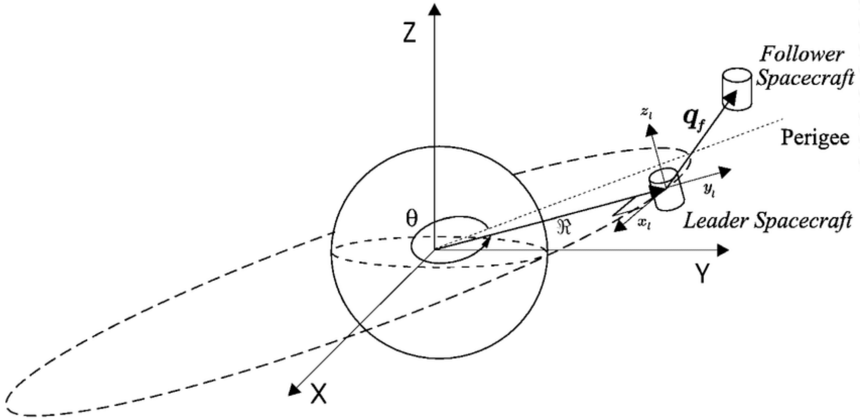
- Relative dynamics of spacecraft in elliptical orbits is time-periodic
- Could approximate with high-order LTI systems
- Forcing freq. changes within one periodic orbit (true anomaly time derivative)

Objectives

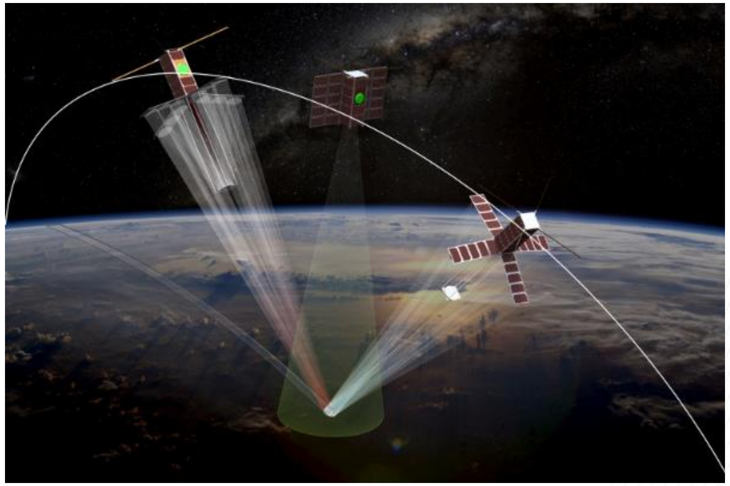
- Extend harmonic decomposition to systems with periodically-varying forcing frequencies
- Perform control design based ofn high-order LTI systems

Interactions

- **P. Singla, R. Melton** (PSU) **M. Lovera** (PoliMi)



Spacecraft Formation Flying



Thank you

Questions?

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